COMPUTER AIDED DESIGN AND MANUFACTURING OF PLATE CAMS

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CERTIFICATE

This is to certify that the work contained in this thesis entitled *Computer Aided Design and Manufacturing of Plate Cams*, by **Vijaykumar S. Kale**, has been carried out under our supervision and that this work has not been submitted elsewhere for award of a degree.

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Abstract

With the rapid developments being made in the field of computer graphics, CAD/ CAM has flourished unboundedly. In the present thesis, development of an integrated system of design and manufacturing of plate cams is presented. A generalised procedure for design of cam for prescribed constraints of pressure angle, radius of curvature and contact stress between cam and follower, analysis and automatic generation of NC codes is presented. A curve fitting procedure generated from the basic definition of cam profile synthesis is used. The design of cam is based on the envelop theory. With the motion specifications as input, cam profile and cutter location data are computed for manufacturing the component on a CNC machine. The NC codes are generated for Denford CNC milling machine with Fanuc controller.

A software has been developed in 'C' and cam profiles designed are displayed in STARBASE graphics on HP workstation operating in 'UNIX' environment.

Contents

List of Tables viii

List of Figures ix

Nomenclature xi

1 Introduction 1

1.1 Conventional Cam Design and Manufacture 2

1.2 Literature Review 4

1.3 Present Work 5

1.4 Organization of Thesis 7

2 Kinematics and Profile Synthesis 8

2.1 Classification of Cam Mechanisms 9

2.1.1 According to Modes of Input/Output Motion 9

2.1.2 According to Arrangement of the Follower 10

2.1.3 According to Motion Events 10

2.2 Types of Motion Curves 13

2.3 Reducing Inertia Forces 18

2.4 Method of Profile Synthesis 20

2.5 A Guideline for Selection of Cam Curves 21

3 Cam Design and Analysis **25**

3.1 Cam Profile Determination 26

3.1.1 Theory of Envelopes 27

3.2 Cutter Coordinates 32

3.3 Pressure Angle 35

3.4 Cam Radius of Curvature 37

3.4.1 Curvature of a Curve 37

3.4.2 Undercutting Phenomenon 39

3.5 Contact Stresses 43

3.5.1 Choice of Cam Materials 44

3.6 Optimum Cam Size 45

4 Implementation **50**

4.1 User Interface 51

4.2 Data input module 52

4.3 Synthesis Module 54

4.4 Cam-Cutter Module 59

4.5 Analysis Module 59

4.6 Optimization Module 65

4.7 Manufacturing Module 66

5 Results and Discussion **67**

5.1 Results 67

5.1.1 Example - 1 67

5.1.2 Example - 2 75

5.2 Discussion 86

6 Conclusions **87**

6.1 Technical Summary 87

6.2 Scope for Future Work 88

References **89**

Appendix **92**

List of Tables

2.1 Comparison of various motion curves 23

2.2 A Guideline for Selection of Cam Curves 24

3.1 Some proven metal combinations 45

3.2 Choice of materials for cams and followers. 46

4.1 Motion curves : velocity factor, acceleration factor 53

4.2 Digit vector representation 54

List of Figures

1.1 Typical cams used in industries 3

1.2 Different phases of design 6

2.1 Six major types of cam-follower arrangement. (a) Disk cam with flat-faced translating follower. (b) Disk cam with radially translating roller follower. (c) Disk cam with offset translating roller follower. (d) Disk cam with oscillating roller follower. (e) Disk cam with eccentric flat-faced oscillating follower. (f) Disk cam with centric flat-faced oscillating follower. 11

2.2 Cam motion events : (a) DRD event (b) DRRD event (c) RRR event. 12

3.1 Cam nomenclature 26

3.2 Envelope of family of curves depending on a parameter. 28

3.3 Envelope of family of circles 29

3.4 Cutter coordinates 33

3.5 Pressure Angle 36

3.6 Curvature of a Curve 38

3.7 Undercutting of cam with roller follower 39

3.8 Undercutting of cam with flat-faced follower 40

4.1 Various steps of Data input module 55

4.2 Help module 56

4.3 Various steps of synthesis module 57

4.4 Synthesis module continue 58

4.5 Block diagram : Cam-cutter Module 60

4.6 Various steps of Analysis module 61

4.7 Block diagram : Pressure Module 63

4.8 Block diagram : Curvature Module 64

5.1 Displacement diagram of example - 1 68

5.2 Displacement curve of example - 1 75

5.3 Velocity curve of example - 1 76

5.4 Acceleration curve of example - 1 76

5.5 Cam profile and Cutter path for example - 1 77

5.6 Incremental positions of Cutter for example - 1 77

5.7 Displacement curve of example - 2 83

5.8 Velocity curve of example - 2 84

5.9 Acceleration curve of example - 2 84

5.10 Cam profile and Cutter path for example - 2 85

5.11 Incremental positions of Cutter for example - 2 85

Nomenclature

$A = A(\theta)$	acceleration equation of the follower follower.
A_i	Initial acceleration of acceleration curve.
A_f	Final acceleration of acceleration curve.
C_v	velocity factor.
C_a	acceleration factor.
$D = D(\theta)$	displacement equation of the follower.
E_1, E_2	Modilii of elasticity of cam & the follower resp.
e	offset of the follower in mm.
L	lift of the follower in mm or degrees.
R_a	cam center to follower pivot distance in mm.
R_b	base circle radius of the cam in mm.
R_c	radius of the cutter in mm.
R_f	radius of the follower roller in mm.
R_0	prime circle radius.
R_r	length of the follower arm in mm.
S	displacement equation of the follower when L in mm.
$S' = \frac{dS}{d\theta}$	first derivative of S with respect to θ.

$S'' = \frac{d^2S}{d\theta^2}$	second derivative of S with respect to θ.
$V = V(\theta)$	velocity equation of the follower.
V_f	Final velocity of velocity curve.
V_i	Initial velocity of velocity curve.
V_{max}	maximum velocity of the follower.
X, Y	rectangular coordinates of the cam profile.
X_c, Y_c	rectangular coordinates of the cutter path.
α	pressure angle.
α_A	Maximum allowable pressure angle (degree).
α_{max}	maximum pressure angle.
β	cam rotation angle for lift L.
μ_1, μ_2	Poissons ratio of cam & follower respectively.
ϕ	displacement equation of the follower when L in degrees.
$\phi' = \frac{d\phi}{d\theta}$	first derivative of ϕ with respect to θ.
$\phi'' = \frac{d^2\phi}{d\theta^2}$	second derivative of ϕ with respect to θ.
ϕ_0	initial angle of the swinging followers.
ρ	radius of curvature.
ρ_c	radius of curvature of the cam curve.
ρ_f	radius of curvature of the follower.
ρ_p	radius of curvature of the pitch curve.
θ	cam rotation angle for follower displacement D.

Chapter 1

Introduction

Computer Aided Design (CAD) and Computer Aided Manufacturing (CAM) which, when properly integrated, lead to Computer Integrated Manufacturing (CIM). CIM is an area in engineering which has made an astonishing progress in the last two decades. Computer Aided Design is a process where the designer and the computer work together to produce an engineering design. As one can not think of CAD without graphics, knowledge of computer graphics is essential to carry out CAD. Computer graphics plays a very important role in communicating design information between the designer and the computer system. Numerically controlled (NC) machining is one of the most important production technology in a modern manufacturing environment. Almost any kind of part can be produced on NC machines.

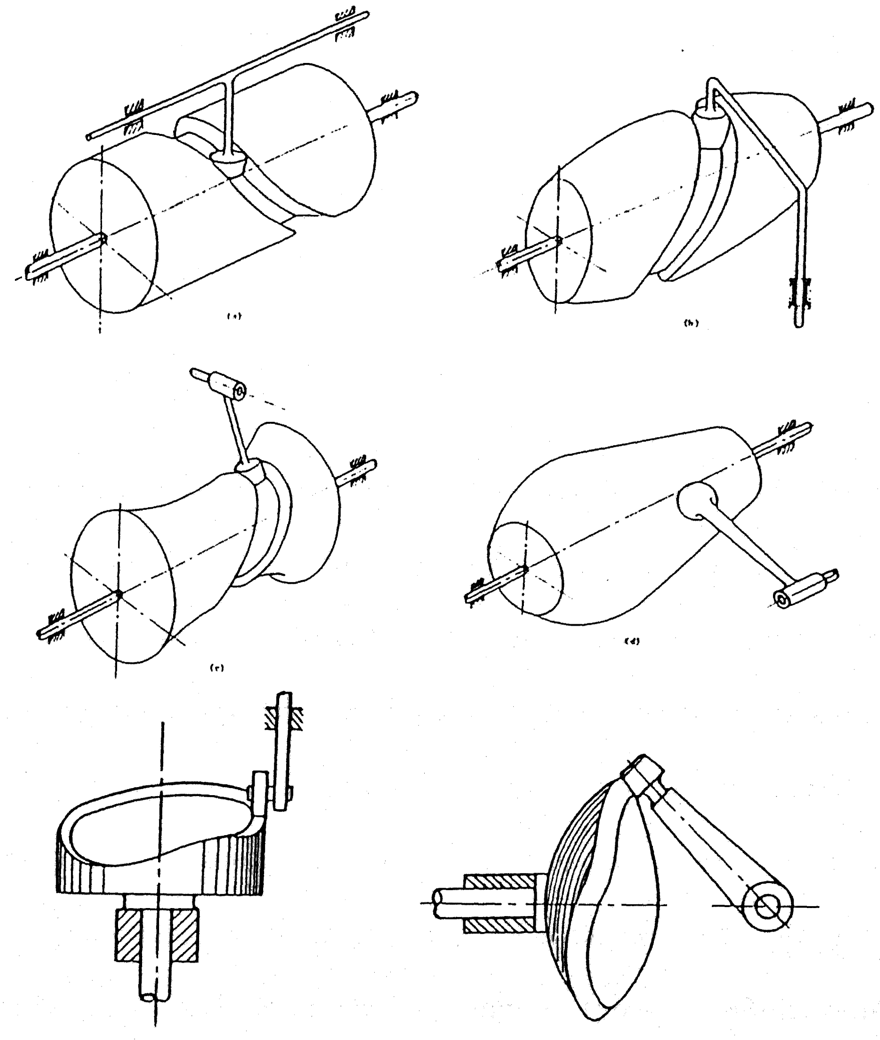
A cam is a mechanical device for transmitting motion to operating element. according to a prescribed motion function. In general, a cam mechanism consists of three three elements the cam, the follower and the frame. Because of widespread applications of automatic machines and instruments, different types of cams are fabricated and used in industries. Among the various cams used in industries, plate cam

is most common, which is cut out of a piece of flate metal or plate. Some examples of cams used in industries are shown in Fig 1.1. Typically cams are used as a component of textile machines, printing presses, food processing machines, internal combustion engine and countless other automatic machines, control systems and devices. This chapter covers traditional ways of design and manufacture of cams and describes established computer based systems of cam design along with their limitations. The motivation of taking up this work are discussed at the end of this chapter.

1.1 Conventional Cam Design and Manufacture

In conventional method of cam design that is graphical method, one needs only to draw a series of circles having their radii equal to the radius of the follower roller and with their centers lying on the pitch curve. The cam profile is nothing but involute of these circles. The cam then is analyzed but only at critical points. However, graphical methods have inherent accuracy limitations and should therefore be used only for relatively slow speed cams. In the design of high speed cam, deviation from the desired curve will result in poor dynamic performance of the cam. Therefore, cam profile and cutter path should be determined analytically. However, analytical cam design is time consuming and tedious process. One can not think of analytical cam design without aid of computers.

In traditional methods cam is manufactured by (i) transferring cam drawing onto a metal plate. Then by combined sawing and filing a master cam is made, (ii) combination of drilling and filing is often used to produce precision point cams. Points on pitch curve are calculated and at these points holes are milled with diameter equal



(a) Cylindrical cam, (b) Globoidal cam (concave), (c) Globoidal cam (convex), (d) Camoid cam, (e) End cam, (f) End cam.

Figure 1.1: Typical cams used in industries

to that of the follower roller. Then by filing desired smoothness is achieved.

In cam design problems, certain prescribed conditions must be fulfilled for a design to be acceptable, and more than one solution usually satisfies the requirements of a particular application. A straightforward procedure may not be available, and an optimum design is only obtained after many trials, each with slightly different input parameters. It is impractical by conventional methods of cam design. Cam manufacturing by conventional methods is also time consuming process and less accurate. This has given an impetus to Computer Aided Design and Manufacturing of cams.

1.2 Literature Review

With the upcoming and widespread use of numerically controlled machine tools a significant effort has been given to the development and implementation of integrated CAD/CAM systems. Liniecki (1975) treated problem of cam size minimization for roller followers under cycloidal, harmonic and eight order polynomial motions. Jansen P. W. (1987) has developed a package for calculation of maximum pressure angle, minimum radius of curvature, maximum contact stress for prescribed motion specification and cam proportions. He attempted only roller followers with few motion curves of DRDR (dwell-rise-dwell-return) types. Only two types of material combinations are considered for stress analysis. His algorithms are based on graphical constructions rather than analytical one. Lin, Chang and Wang (1988) have developed an integrated system of cam design and manufacturing. Their work is limited to only calculation of cam profile and cutter coordinates. They considered only cycloidal, simple harmonic motion and 8^{th} degree polynomial for synthesis.

Gal-Tzur, Shpitalni and Malkin (1989) have developed an integrated system of cam design and manufacturing for roller followers. They presented a methodology for machining of cam by CNC grinding. They analyzed cam only for minimum radius of curvature. K. Yoon and S. Rao (1993) treated cubic splines for cam motion synthesis. However, this creates limitations as the user should have the knowledge of mathematics and behaviour of splines under consideration. Angels, Lopez-Cajun and Saha (1994) presented optimum design of cam with translating flat-faced followers under curvature constraints. They used spline function for synthesis.

1.3 Present Work

Design, analysis and actual manufacturing are three broad steps in any product development process. These three aspects are always interlinked. Fig. 1.2 shows different phases of design. All these aspects are treated in an integrated system of cam design and manufacturing.

The objective of the present work is to develop an integrated system of cam design and manufacturing with computer to design, analyze and optimize the cam and produce the NC code for manufacturing; with sufficient flexibility given to user to change the various parameters and observe the results.

One of the main requirements for the design of cam is to obtain a set of motion curves from which any complex acceleration pattern can be obtained by 'methode of profile synthesis'. To achieve this requirement 16 types of motion curves for rise and return are considered for synthesis. With the motion function and parameters related to follower type as the essential input, the presently developed package designs the

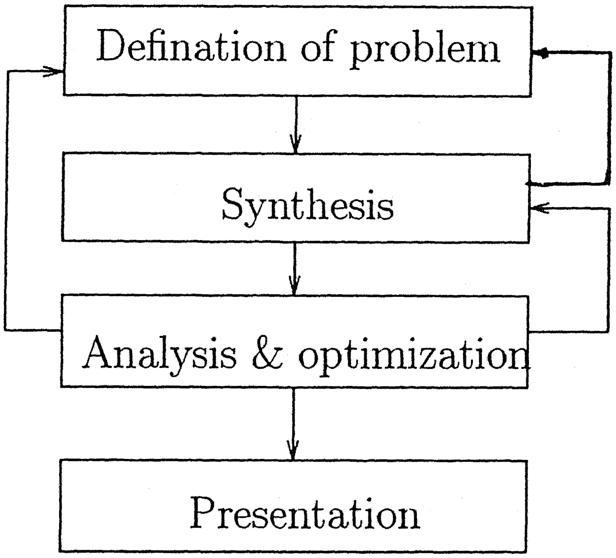


Figure 1.2: Different phases of design

plate cams interactively. To obtain velocity match equations and acceleration match equations in the combined curve fitting process, a methodology is presented based on digit vectors. The displacement, velocity and acceleration digrams are displayed on screen. The cam design is based on the theory of envelopes. The cutter location data for manufacturing can be calculated in rectangular coordinate system or polar coordinate system. The system displays cam profile and cutter path on the screen. The designed cam is analyzed for maximum pressure angle, minimum radius of curvature and contact stress between cam and follower. Eight types of cam-follower metal combinations are considered for stress analysis. The present system suggests optimum cam proportions for prescribed pressure angle and motion specifications. To prepare a NC part program is a lengthy process. The cutter location data produced in the design stage is directly used for manufacturing. The system automatically generates the NC code to fabricate cam on NC milling or NC grinding machine.

1.4 Organization of Thesis

Chapter 2 covers the types of motion events, types of basic motion curves, their characteristics, comparison and selection. The 'method of profile synthesis' is also described. Chapter 3 describes theory of envelopes, cam profile calculations, cutter coordinate calculations, analysis of cam under constraints of pressure angle, radius of curvature and contact stress between cam and follower. Optimization of cam follower parameters is also given.

Chapter 4 deals with the implementation part of the problem. Detailed information about various modules used in the program are discussed. Block digrams illustrating the modules are presented. Integration of design and manufacturing is given at the end of this chapter. Chapter 5 gives an overview about the performance of the system. The results obtained from a trial run of the system are presented. In Chapter 6 a technical summary of the present work and scope for future work are presented.

Chapter 2

Kinematics and Profile Synthesis

As mentioned earlier a *cam* is a mechanical component of a machine that is used to transmit motion to another component, called the *follower*, through a prescribed motion program by direct contact.

A cam mechanism consists of three elements the *cam*, the *follower* and the *frame*. The follower is in direct contact with the cam. The cam may be of various shapes. The follower system includes all of the elements to which motion is imparted by the cam. The frame of the machine supports the bearing surfaces for the cam and for the follower.

The *cam* mechanism is a *versatile* one. It is used to transform a rotary motion into a translating or oscillating motion. It can be designed to produce almost unlimited types of motions in the follower by using suitable types of cams. On certain occasions, it is also used to transform one translating or oscillating motion into a different translating or oscillating motions. All translational or oscillatory input/output motions can belong to any of the following categories according to their viability in time.

- Nonuniform motion - Programmed variable speed.

- Intermittent motion - Cyclic interval of dwells.
- Reversing - Cyclic change in motion direction.

Cams are used in wide variety of automatic machines and instruments. Typical examples of their usage include textile machinaries, mechanical and electronic computers, printing presses, packaging machines, internal combustion engines, food processing machines, can making machinery, wire forming machines, computing mechanisms and countless other automatic machines, control systems and devices. The cam mechanism is indeed a very important component in *modern mechanization.*

2.1 Classification of Cam Mechanisms

We can classify cam mechanisms according to the modes of input/ output motion, the configuration and the arrangement of follower. We can also classify cams according to different types of motion events of the follower.

2.1.1 According to Modes of Input/Output Motion

- Rotating cam - translating follower.
- Rotating cam - swinging or oscillating follower.
- Translating cam - translating follower.
- Stationary cam - translating follower.
- Stationary cam - rotating follower.

2.1.2 According to Arrangement of the Follower

- Cam with flat-faced translating follower.
- Cam with radially translating roller follower.
- Cam with offset translating roller follower.
- Cam with swinging roller follower.
- Cam with eccentric flat-faced oscillating follower.
- Cam with centric flat-faced oscillating follower.

Fig 2.1 shows types of cam and follower arrangements.

In all the cases the follower must be constrained to follow the cam. This may be done by means of a

- Gravity constraint - the weight of the follower system itself is sufficient to maintain contact with the cam.
- Spring constraint - the spring must be properly designed to maintain contact.
- Positive mechanical constraint - a groove, in which the follower is retiained maintains positive action.

2.1.3 According to Motion Events

When cam turns through one motion cycle, the follower executes a series of events consisting of rises, dwells and returns. Rise is the motion of the follower away from the cam center, dwell is the motion during which the follower is at rest; and return is

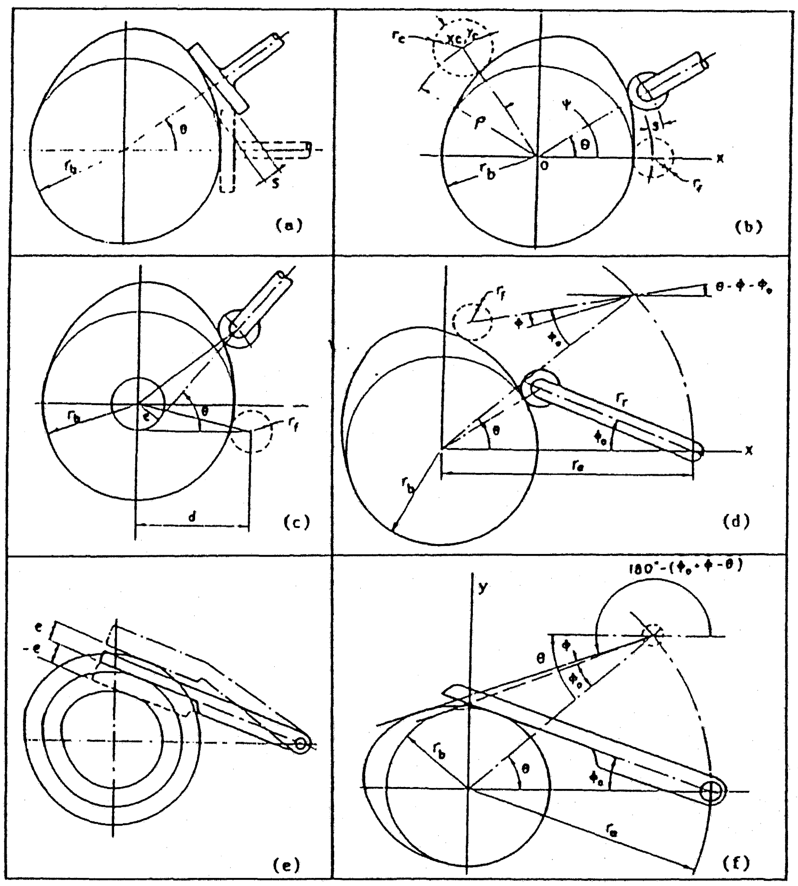


Figure 2.1: Six major types of cam-follower arrangement. (a) Disk cam with flat-faced translating follower. (b) Disk cam with radially translating roller follower. (c) Disk cam with offset translating roller follower. (d) Disk cam with oscillating roller follower. (e) Disk cam with eccentric flat-faced oscillating follower. (f) Disk cam with centric flat-faced oscillating follower.

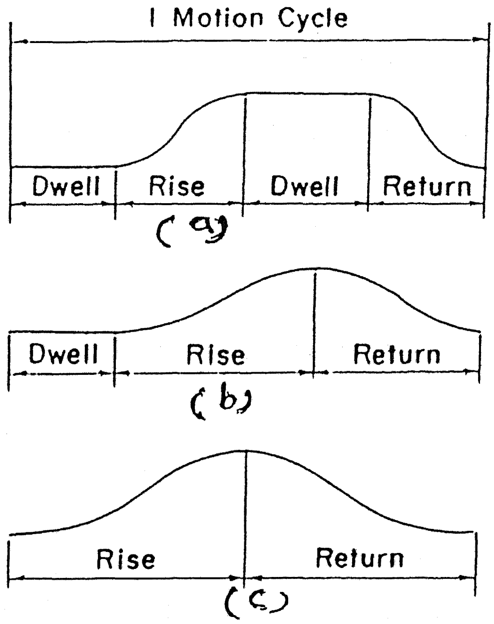


Figure 2.2: Cam motion events : (a) DRD event (b) DRRD event (c) RRR event.

the motion of the follower toward the cam center. The following events of follower's motion are common.

DRDR (*dwell-rise-dwell-return*) : It has dwell at the beginning and the end of rise and return. We could just as well consider it as dwell-return-dwell-rise, the meaning being the same.

DRRD (*dwell-rise-return-dwell*) : There is no dwell between rise and return. We could just as well consider it as dwell-return-rise-dwell.

RRR (*rise-return-rise*) : In this case there is no dwell.

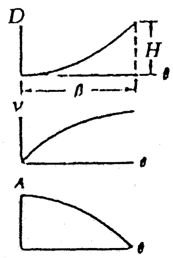
Fig. 2.2 shows basic events of the follower's motion.

2.2 Types of Motion Curves

There are many follower motions that can be used for rises and the returns. The mathematical expressions and corresponding characteristics for rise of 16 types of curves considered for synthesis are given [8] below.

In the following expressions D is displacement, V is velocity and A is acceleration of the follower. H is lift is lift of the follower. β is cam ortation angle for lift H and θ is cam rotation angle for corresponding displacement D.

(1) Half Simple Harmonic Motion (acceleration)

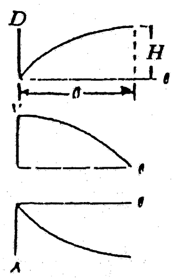


$$D = \frac{H}{2}\ (1 - \cos\ (\frac{\pi}{2}\frac{\theta}{\beta}))$$

$$V = \frac{\pi}{2}\frac{H}{\beta}\ (\sin\ (\frac{\pi}{2}\frac{\theta}{\beta}))$$

$$A = \frac{\pi^2}{4}\frac{H}{\beta^2}\ (\cos\ (\frac{\pi}{2}\frac{\theta}{\beta}))$$

(2) Half Simple Harmonic Motion (decceleration)

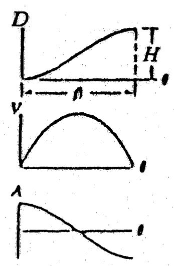


$$D = H\ (\sin\ (\frac{\pi}{2}\frac{\theta}{\beta}))$$

$$V = \frac{\pi}{2}\frac{H}{\beta}\ (\cos\ (\frac{\pi}{2}\frac{\theta}{\beta}))$$

$$A = -\frac{\pi^2 H}{4\ \beta^2}(\sin\ (\frac{\pi}{2}\frac{\theta}{\beta}))$$

(3) Simple Harmonic Motion

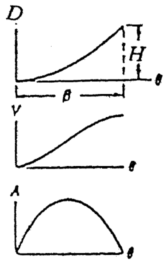


$$D = H\ (\sin\ (\frac{\pi\ \theta}{\beta}))$$

$$V = \frac{\pi}{2}\frac{H}{\beta}(\sin\ (\frac{\pi\ \theta}{\beta}))$$

$$A = \frac{\pi^2}{2}\frac{H}{\beta^2}(\cos\ (\frac{\pi\ \theta}{\beta}))$$

(4) Half Cycloidal (acceleration)

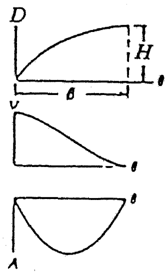


$$D = H\ (\tfrac{\theta}{\beta} - \tfrac{1}{\pi} \sin\ (\tfrac{\pi\ \theta}{\beta}))$$

$$V = \tfrac{H}{\beta}\ (1 - \cos\ (\tfrac{\pi\ \theta}{\beta}))$$

$$A = \tfrac{\pi\ H}{\beta^2}(\sin\ (\tfrac{\pi\ \theta}{\beta}))$$

(5) Half Cycloidal (decceleration)

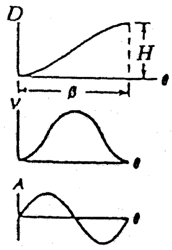


$$D = H\ (\tfrac{\theta}{\beta} + \tfrac{1}{\pi} \sin\ (\tfrac{\pi\ \theta}{\beta}))$$

$$V = \tfrac{H}{\beta}\ (1 + \cos\ (\tfrac{\pi\ \theta}{\beta}))$$

$$A = -\tfrac{\pi\ H}{\beta^2}(\sin\ (\tfrac{\pi\ \theta}{\beta}))$$

(6) Cycloidal

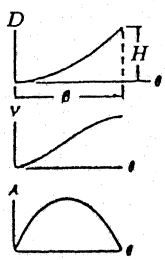


$$D = H\ (\tfrac{\theta}{\beta} - \tfrac{1}{2\ \pi} \sin\ (\tfrac{2\ \pi\ \theta}{\beta}))$$

$$V = \tfrac{H}{\beta}\ (1 - \cos\ (\tfrac{2\ \pi\ \theta}{\beta}))$$

$$A = \tfrac{2\ \pi\ H}{\beta^2}(\sin\ (\tfrac{2\ \pi\ \theta}{\beta}))$$

(7) Half 3-4-5 Polynomial (acceleration)

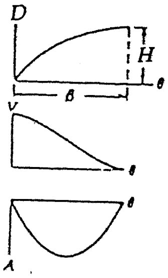


$$D = \tfrac{H}{4}\ (10\ (\tfrac{\theta}{\beta})^3 - 7.5\ (\tfrac{\theta}{\beta})^4 + 1.5\ (\tfrac{\theta}{\beta})^5)$$

$$V = \tfrac{H}{4\ \beta}\ (30\ (\tfrac{\theta}{\beta})^2 - 30\ (\tfrac{\theta}{\beta})^3 + 7.5\ (\tfrac{\theta}{\beta})^4)$$

$$A = \tfrac{H}{4\ \beta^2}\ (60\ (\tfrac{\theta}{\beta}) - 90\ (\tfrac{\theta}{\beta})^2 + 30\ (\tfrac{\theta}{\beta})^3)$$

(8) Half 3-4-5 Polynomial (decceleration)

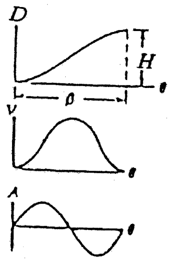


$$D = \frac{H}{4}\ (7.5\ (\tfrac{\theta}{\beta}) - 5\ (\tfrac{\theta}{\beta})^3 + 1.5\ (\tfrac{\theta}{\beta})^5)$$

$$V = \frac{H}{4\ \beta}\ (7.5\ - 15\ (\tfrac{\theta}{\beta})^2 + 7.5\ (\tfrac{\theta}{\beta})^4)$$

$$A = \frac{H}{4\ \beta^2}\ (-30\ (\tfrac{\theta}{\beta}) + 30\ (\tfrac{\theta}{\beta})^3)$$

(9) 3-4-5 Polynomial

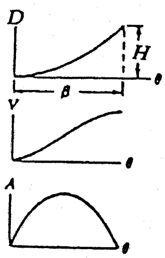


$$D = H\ (10\ (\tfrac{\theta}{\beta})^3 - 15\ (\tfrac{\theta}{\beta})^4 + 6\ (\tfrac{\theta}{\beta})^5)$$

$$V = \frac{H}{\beta}\ (30\ (\tfrac{\theta}{\beta})^2 - 60\ (\tfrac{\theta}{\beta})^3 + 30\ (\tfrac{\theta}{\beta})^4)$$

$$A = \frac{H}{\beta^2}\ (60\ (\tfrac{\theta}{\beta}) - 180\ (\tfrac{\theta}{\beta})^2 + 120\ (\tfrac{\theta}{\beta})^3)$$

(10) 4-5-6-7 Polynomial (acceleration)

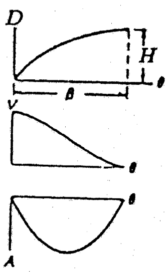


$$D = \frac{H}{8}\ (35\ (\tfrac{\theta}{\beta})^4 - 42\ (\tfrac{\theta}{\beta})^5 + 17.5\ (\tfrac{\theta}{\beta})^6 - 2.5\ (\tfrac{\theta}{\beta})^7)$$

$$V = \frac{H}{8\ \beta}\ (140\ (\tfrac{\theta}{\beta})^3 - 210\ (\tfrac{\theta}{\beta})^4 + 105\ (\tfrac{\theta}{\beta})^5 - \tfrac{35}{2}\ (\tfrac{\theta}{\beta})^6)$$

$$A = \frac{H}{8\ \beta^2}\ (420\ (\tfrac{\theta}{\beta})^2 - 840\ (\tfrac{\theta}{\beta})^3 + 525\ (\tfrac{\theta}{\beta})^4 - 105\ (\tfrac{\theta}{\beta})^5)$$

(11) 4-5-6-7 Polynomial (decceleration)

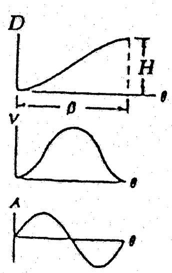


$$D = \frac{H}{8}\ (\tfrac{35}{2}\ (\tfrac{\theta}{\beta}) - 17.5\ (\tfrac{\theta}{\beta})^3 + 10.5\ (\tfrac{\theta}{\beta})^5 - 2.5\ (\tfrac{\theta}{\beta})^7)$$

$$V = \frac{H}{8\ \beta}\ (17.5\ (\tfrac{\theta}{\beta}) - 57.5\ (\tfrac{\theta}{\beta})^2 + 57.5\ (\tfrac{\theta}{\beta})^4 - 17.5\ (\tfrac{\theta}{\beta})^6)$$

$$A = \frac{H}{8\ \beta^2}\ (-105\ (\tfrac{\theta}{\beta}) + 210\ (\tfrac{\theta}{\beta})^3 - 105\ (\tfrac{\theta}{\beta})^5)$$

(12) 4-5-6-7 Polynomial



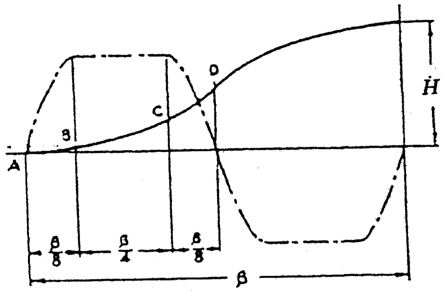
$$D = H\ (35\ (\tfrac{\theta}{\beta})^4 - 84\ (\tfrac{\theta}{\beta})^5 + 70\ (\tfrac{\theta}{\beta})^6 - 20\ (\tfrac{\theta}{\beta})^7)$$

$$V = H\ (140\ (\tfrac{\theta}{\beta})^3 - 420\ (\tfrac{\theta}{\beta})^4 + 420\ (\tfrac{\theta}{\beta})^5 - 140\ (\tfrac{\theta}{\beta})^6)$$

$A = H\ (420\ (\frac{\theta}{\beta})^2 - 1680\ (\frac{\theta}{\beta})^3 + 2100\ (\frac{\theta}{\beta})^4 - 840\ (\frac{\theta}{\beta})^5)$

(13) Modified Trapezoidal Motion

for $0 \leq \frac{\theta}{\beta} \leq 1/8$



$D = 0.09724612\ H\ (4\ \frac{\theta}{\beta} - (\sin\ (4\ \pi\ \frac{\theta}{\beta}))\ /\pi)$

$V = 0.3889845\ (\frac{H}{\beta})\ (1 - \cos\ (4\ \pi\ \frac{\theta}{\beta}))$

$A = 4.88124\ (\frac{H}{\beta^2})\ \sin\ (4\ \pi\ \frac{\theta}{\beta}))$

for $1/8 \leq \frac{\theta}{\beta} \leq 3/8$

$D = H\ (2.44406184\ (\frac{\theta}{\beta})^2 - 0.22203097\ \frac{\theta}{\beta} + 0.00723407)$

$V = \frac{H}{\beta}\ (4.888124\ \frac{\theta}{\beta} - 0.22203097)$

$A = 4.88124\ (\frac{H}{\beta^2})$

for $3/8 \leq \frac{\theta}{\beta} \leq 1/2$

$D = H\ (1.6110154\ \frac{\theta}{\beta} - 0.0309544\ \sin\ (4\ \pi\ \frac{\theta}{\beta} - \pi) - 0.305077)$

$V = \frac{H}{\beta}\ (1.6110154\ - 0.3889845\ \cos\ (4\ \pi\ \frac{\theta}{\beta} - \pi))$

$A = 4.88124\ \frac{H}{\beta^2}\ \sin\ (4\ \pi\ \frac{\theta}{\beta} - \pi)$

for $1/2 \leq \frac{\theta}{\beta} \leq 5/8$

$D = H\ (1.6110154\ \frac{\theta}{\beta} + 0.0309544\ \sin\ (4\ \pi\ \frac{\theta}{\beta} - 2\ \pi) - 0.305077)$

$V = \frac{H}{\beta}\ (1.6110154\ + 0.3889845\ \cos\ (4\ \pi\ \frac{\theta}{\beta} - 2\ \pi))$

$A = 4.88124\ \frac{H}{\beta^2}\ \sin\ (-4\ \pi\ \frac{\theta}{\beta} - 2\ \pi)$

for $5/8 \leq \frac{\theta}{\beta} \leq 7/8$

$$D = H\ (-\ 2.44406184\ (\tfrac{\theta}{\beta})^2 + 4.6660917\ \tfrac{\theta}{\beta} - 1.2292648)$$

$$V = \tfrac{H}{\beta}\ (4.6660917\ \ - 4.6660917\ \tfrac{\theta}{\beta})$$

$$A = -\ 4.88124\ (\tfrac{H}{\beta^2})$$

for $7/8 \leq \frac{\theta}{\beta} \leq 1$

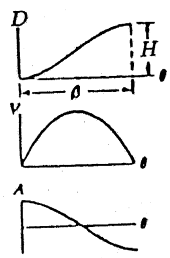
$$D = H\ (0.38898845\ \tfrac{\theta}{\beta} + 0.0309544\ \sin\ (4\ \pi\ \tfrac{\theta}{\beta} - 3\ \pi) + 0.6110154)$$

$$V = \tfrac{H}{\beta}\ (0.3889845\ \ + 0.3889845\ \cos\ (4\ \pi\ \tfrac{\theta}{\beta} - 3\ \pi))$$

$$A = -\ 4.88124\ \tfrac{H}{\beta^2}\ \sin\ (4\ \pi\ \tfrac{\theta}{\beta} - 3\ \pi)$$

(14) 5^{th} degree Polynomial

$$D = H\ (2.5\ (\tfrac{\theta}{\beta})^2 - 2.5\ (\tfrac{\theta}{\beta})^4 + (\tfrac{\theta}{\beta})^5)$$



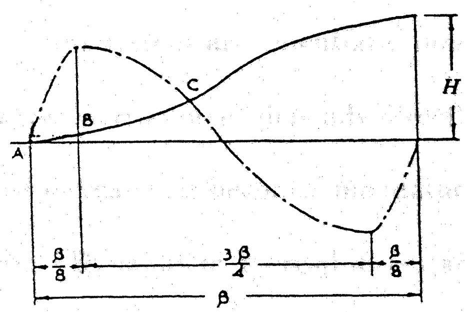
$$V = \tfrac{H}{\beta}\ (5\ (\tfrac{\theta}{\beta}) - 10\ (\tfrac{\theta}{\beta})^3 + 5\ (\tfrac{\theta}{\beta})^4)$$

$$A = \tfrac{H}{\beta^2}\ (5\ \ - 30\ (\tfrac{\theta}{\beta})^2 + 20\ (\tfrac{\theta}{\beta})^3)$$

(15) Modified Sine

for $0 \leq \frac{\theta}{\beta} \leq 1/8$

$$D = H\ (0.43989\ \tfrac{\theta}{\beta} - 0.035014\ \sin\ (4\ \pi\ \tfrac{\theta}{\beta}))$$



$$V = 0.43989\ \tfrac{H}{\beta}\ (1 - \cos\ (4\ \pi\ \tfrac{\theta}{\beta}))$$

$$A = 5.52794\ \tfrac{H}{\beta^2}\ \sin\ (4\ \pi\ \tfrac{\theta}{\beta}))$$

for $1/8 \leq \frac{\theta}{\beta} \leq 7/8$

$D = H\ (0.28005\ + 0.43989\ \frac{\theta}{\beta} - 0.0315055\ \cos\ (\frac{4}{3}\ \pi\ \frac{\theta}{\beta}\ -\ \frac{\pi}{6}))$

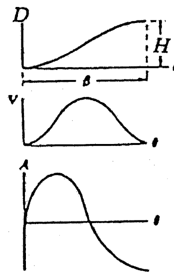
$V = \frac{H}{\beta}\ (0.43989\ + 1.31976\ \sin\ (\frac{4}{3}\ \pi\ \frac{\theta}{\beta}\ -\ \frac{\pi}{6}))$

$A = 5.52794\ \frac{H}{\beta^2}\ \cos\ (\frac{4}{3}\ \pi\ \frac{\theta}{\beta}\ -\ \frac{\pi}{6}))$

for $7/8 \leq \frac{\theta}{\beta} \leq 1$

$D = H\ (0.56010\ +\ 0.43989\ \frac{\theta}{\beta} - 0.035014\ \sin\ (4\ \pi\ \frac{\theta}{\beta}\ -\ 2\ \pi))$

$V = 0.43989\ \frac{H}{\beta}\ (1 - \cos\ (4\ \pi\ \frac{\theta}{\beta}\ -\ 2\ \pi))$



$A = 5.52794\ \frac{H}{\beta^2}\ \sin\ (4\ \pi\ \frac{\theta}{\beta}\ -\ 2\ \pi))$

(16) 8^{th} degree Polynomial

$D = H\ (6.09755\ (\frac{\theta}{\beta})^3 - 20.78084\ (\frac{\theta}{\beta})^5 + 26.73155\ (\frac{\theta}{\beta})^6 - 13.60965\ (\frac{\theta}{\beta})^7 + 2.56095\ \frac{\theta}{\beta})^8)$

$V = \frac{H}{\beta}\ (18.29265\ (\frac{\theta}{\beta})^2 - 103.90200\ (\frac{\theta}{\beta})^4 + 160.38930\ (\frac{\theta}{\beta})^5 - 95.26755\ (\frac{\theta}{\beta})^6 + 20.48760\ \frac{\theta}{\beta})^7)$

$A = \frac{H}{\beta^2}\ (36.58530\ (\frac{\theta}{\beta}) - 415.60800\ (\frac{\theta}{\beta})^3 + 801.94650\ (\frac{\theta}{\beta})^4 - 571.60530\ (\frac{\theta}{\beta})^5 + 143.41320\ \frac{\theta}{\beta})^6)$

The return curves are mirror images of the rise curves. To obtain expressions for these curves replace θ by $(\beta - \theta)$

2.3. Reducing Inertia Forces

The work, friction, spring and gravity forces of a cam system are essentially independent of the motion provided by the cam. However inertia forces depends directly on the cam profile, therefore as the operating speed increases, it becomes mandatory to select profiles on the basis of these inertia forces. Dynamic or inertial forces are created by the necessity to move the machine elements. Moving machine parts are

subjected to acceleration. During each stroke the resisting forces vary between zero and positive and negative maximum. The variation of forces between zero and positive and negative maximum is the cause of vibrations of moving parts, which creates additional stresses.

The inertia or D'Alembert forces are a product of the mass and the acceleration of the follower system. For low speeds, the inertia load is low and will cause no appreciable deflection of component. At high speeds, however, the inertia load will increase. The cam mechanism, loaded by the increasing inertia forces is prone to deflection and creates vibrations, which produce vibratory stresses in the follower system and remain after the cam rise is completed. These stresses may magnify the basic stresses caused by the accelerating forces generated by the cam profile, resulting in fatigue between cam and follower.

In case of movements created by cams, the designer can select a desirable acceleration diagram and, therefore can control dynamic forces to a certain extent. The choice of optimum profile depends, to a considerable extents, on the corresponding dynamic performance of the profile. The velocity and acceleration factors plays important role in this regards.

It is possible to have the maximum acceleration as low as possible. Instantaneous changes of acceleration should be avoided. The velocity factor C_v and the acceleration factor C_a facilitates the designer to compare various motion curves. Velocity factor and the acceleration factor are the coefficients of maximum velocity and maximum acceleration respectively. A lower velocity factor means lower maximum velocity, and a lower kinetic energy of the moving mass $= m.V_{max}^2$. Also lower velocity factor means

lower maximum slope of the cam, so the cam can be made smaller. The knowledge of C_a and C_v permits to select the desirable acceleration diagram. This brings about the careful *matching* and *blending* together of composite profiles. This technique is called as "Methode of Profile Synthesis" [8].

2.4 Method of Profile Synthesis

The motion curves described in the previous section are used because of their simplicity of construction and ease of analysis. In situations where these simple curves are inadequate such as the requiring of specific velocities, intermediate displacements or accelerations and motion specification for unsymmetrical rise and fall, combination of motions is a good solution.

The fundamental conditions to be satisfied for combining curves are

(1) The sum of the displacements during each of the various motions is equal to the total stroke, H

$$S_{r1} + S_{r2} + \cdots = S_{f1} + S_{f2} + \cdots = H$$

where S_{r1} – displacement for the first rise.

S_{r2} – displacement for the second rise, etc.

S_{f1} – displacement for the first fall.

S_{f2} – displacement for the second fall, etc.

(2) The sum of the angle turned through by the cam during each of the various motions is equal to the total cam angle beta (β)

$$\beta_1 + \beta_2 + \cdots = \beta$$

where β_i is the rotation angle during the ith segment.

(3) It is necessary that velocities of all the curves at the junction be equal. The required equations can be formulated as

$$C_v(i) \;.\; S_i/\beta_i \;=\; C_v(i+1) \;.\; S_{i+1}/\beta_{i+1}$$

where $C_v(i)$ and $C_v(i+1)$ are velocity factors of the ith and $(i+1)$th motion curves.

S_i and S_{i+1} are displacements of the ith and $(i+1)$th motion curves.

(4) An advanced condition for high speed action requires that the acceleration of all the curves at the junction be equal

The acceleration match equation is

$$C_a(i) \;.\; S_i/\beta_i^2 \;=\; C_a(i+1) \;.\; S_{i+1}/\beta_{i+1}^2$$

where $C_a(i)$ and $C_a(i+1)$ are the acceleration factors of the ith and $(i+1)$th motion curves.

Bounded by above constraints, the rotation angle and displacement of each block can be calculated.

2.5 A Guideline for Selection of Cam Curves

The selection of cam curves depends on the events such as DRD, DRRD etc. In addition to this the designer should have the knowledge of C_v and C_a from the point of view of inertia forces.

Among the curves, *Cycloidal, 3-4-5 Polynomial, 4-5-6-7 Polynomial* , *Modified Trapezoidal, Modified Sine* are suitable for *DRD* type of event, since acceleration at

the start and end of these curves are zero. Therefore these curves can be matched with dwells.

Simple Harmonic Motion is particularly useful for *RRR* event, where acceleration at both the start and finish can be matched with adjacent profiles. *Half-Harmonic Motion* can be used where *constant velocity* rise follows acceleration. Alternatively, the *Half-Harmonic* curve could be coupled to a *Half-Cycloidal* or to a 8^{th} *degree Polynomial.*

8^{th} *degree Polynomial* and 5^{th} *degree Polynomial* are suitable for *DRRD* event. Acceleration at the end of rise should be matched with the acceleration at the start of next curve. Zero acceleration at start can be coupled to a Dwell.

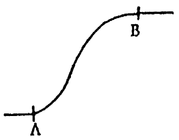
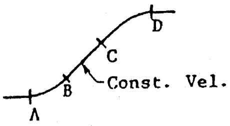
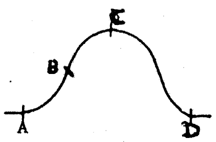
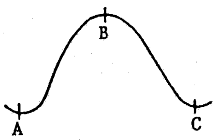
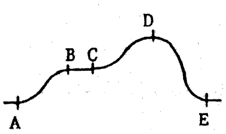
Table 2.1 shows motion curves with their corresponding velocity factor, acceleration factor and speed applications.

Table 2.2 shows basic events, required boundary conditions and recommended types of motion curves.

Table 2.1: Comparison of various motion curves

Motion Curves	Velocity Factor	Acceleration Factor	Speed Application
Half Cycloidal	2.0	π	High speed
Cycloidal	2.0	2π	High speed
Half Simple Harmonic Motion	$\pi/2$	2.4560	High speed
Simple Harmonic Motion	$\pi/2$	4.9300	High speed
Half 3-4-5 Polynomial	1.890	2.8850	High speed
3-4-5 Polynomial	1.890	5.7700	High speed
Half 4-5-6-7 Polynomial	2.190	3.7600	Medium speed
4-5-6-7 Polynomial	2.190	7.5200	Medium speed
Modified Trapezoidal	2.0	4.8900	High speed
Modified Sine	1.76	5.5300	High speed
5^{th} degree Polynomial	1.5625	5.0000	High speed
8^{th} degree Polynomial	—	5.2683	High speed

Table 2.2: A Guideline for Selection of Cam Curves

Displacement Curve	Required Boundary Conditions	Recommended Types of Motions	
 DRD	Vel. at A, B = 0 Acceleration at A, B = 0	Modi. Trapezoidal, cyclodial, Modi. Sine, 3-4-5 Poly., 4-5-6-7 Poly.	
 DR-CV-DR	Acceleration at A, B, C, D = 0 Vel. at A, D = 0 Velocity at B = Velocity at C	AB	CD
		Half_Cycloidal. Half_3-4-5 Poly. Half_4-5-6-7 Poly.	Half_Cycloidal. Half_3-4-5 Poly. Half_4-5-6-7 Poly.
		BC – Constant Vel.	
 DRRD	Acceleration at A, B, D = 0 Accel. match at C Velocity at A, C, D = 0 Vel. match at B	AB	CD
		Half_Cycloidal. Half_3-4-5 Poly. Half_4-5-6-7 Poly.	8th degree Poly. 5th degree Poly.
		BC – Half_SHM	
 RRR	Acceleration match At A, B, C Velocity at A, B, C = 0	AB	BC
		Half_SHM.	Half_SHM.
 Combination	Acceleration at A, B, C, E = 0 Velocity at A, B, C, D, E = 0 Vel. match at D	AB	CD
		Cycloidal. 3-4-5 Poly. 4-5-6-7 Poly. Modi. Trapezoidal. Modified Sine.	5^{th} degree Poly. 8^{th} degree Poly.
		DE – 8^{th} Poly., 5^{th} Poly.	

Chapter 3

Cam Design and Analysis

In any cam design problem the first decision always relates to motion specifications. It is necessary to develop a complete timing diagram for the machine, showing the displacement of the mechanism and its proper spacing and place in the time cycle. The cam design is not merely a task of getting cam profile and cutter profile data. For healthy working of the machine it should follow the basic constrains. The basic limiting requirements are pressure angle, radius of curvature and finally the contact pressure between cam and follower. Selection of cam dimensional parameters such as the base circle radius and the follower offset is normally made at an early stage of cam design to satisfy one or more of the above constraints. So the designer should check the above constraints after each design stage, and make the changes in the input accordingly. Cam design problem is a good example of synthesis. In fact, designing a cam mechanism from the desired motion is an application of synthesis that can be solved every time. Fig 3.1 shows cam nomenclature.

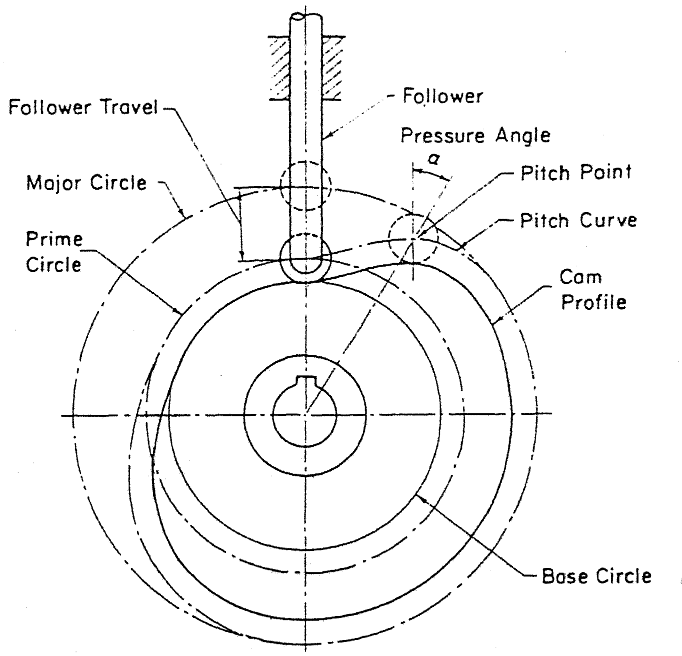


Figure 3.1: Cam nomenclature

3.1 Cam Profile Determination

Conventional method of cam design have inherent accuracy limitations and should therefore be used only for relatively slow speed cams. Also pitch curve determined by graphical layout is not adequate, especially when the cam is manufactured by incremental cutting process or by a numerically controlled milling machine. Therefore the cam profile or the cutter path should be determined analytically.

The theory of envelopes is a powerful analytical tool. The cam profile and cutter coordinate equations are derived for six major types of cam follower arrangements, by [1] using theory of envelopes. The following coordinate systems have been used for deriving cam profiles and cutter coordinate equations.

- Polar coordinates.
- Rectangular coordinates.

The major types of cam follower arrangements shown in fig 2.1 are quoted below.

1. Cam with radially translating roller follower.

2. Cam with translating offset roller follower.

3. Cam with swinging roller follower.

4. Cam with translating flat faced follower.

5. Cam with swinging centric flat faced follower.

6. Cam with swinging eccentric flat faced follower.

3.1.1 Theory of Envelopes

Suppose $S\gamma_c$ is a family of smooth curves on a surface, depending on parameter δ. A smooth curve γ is called an envelope of the family S if

(a) for every point of the curve γ it is possible to give a curve γ_c of the family that is tangent to the curve γ at this point.

(b) for every curve γ_c of the family it is possible to give a point on the curve γ at which curve γ_c is tangent to γ, and

(c) no curve of the family has a segment in common with the curve γ (see Fig 3.2).

Linearly moving circles :

As an example of envelope theory, consider the equation

$$(x-c)^2 + (y)^2 - 1 = 0 \tag{3.1}$$

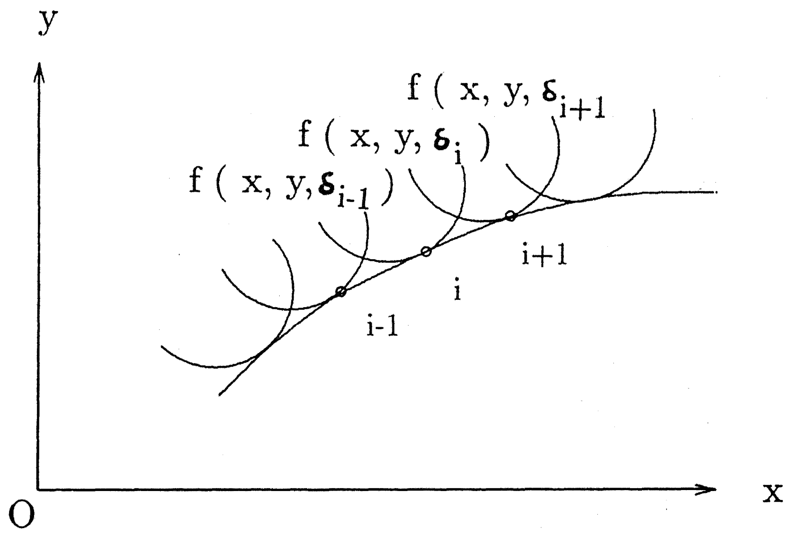


Figure 3.2: Envelope of family of curves depending on a parameter.

equation 3.1 represents a circle of radius 1 located with center at $x = c$, $y = 0$.

As c is varied, a series of circles are determined.

The family of circles are governed by parameter c.

Equation 3.1 can be rewritten as

$$f(x, y, c) = 0 \tag{3.2}$$

The slope or gradient of any member of the family of equation 3.2 is

$$\frac{dy}{dx} = -\frac{\partial f/\partial x}{\partial f/\partial y} \tag{3.3}$$

or

$$\frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = 0 \tag{3.4}$$

This may be written as

$$\frac{\partial f}{\partial x}\frac{dx}{dc} + \frac{\partial f}{\partial y}\frac{dy}{dc} = 0 \tag{3.5}$$

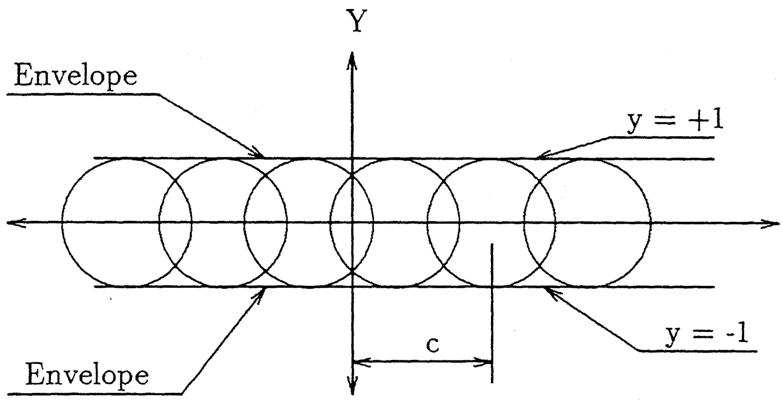


Figure 3.3: Envelope of family of circles

This slope relation holds true for any member of family.

If another curve (envelope) is tangent to the member of the family at a single point it should satisfy equation 3.5

The total differentiation of equation 3.2 is

$$\frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial c}dc = 0$$

or

$$\frac{\partial f}{\partial x}\frac{dx}{dc} + \frac{\partial f}{\partial y}\frac{dy}{dc} + \frac{\partial f}{\partial c} = 0 \tag{3.6}$$

From equation 3.5 and equation 3.6 the general equation of envelope is

$$\frac{\partial f(x,y,c)}{\partial c} = 0 \tag{3.7}$$

The envelope may be determined by eliminating the parameter c in equation 3.7 or by obtaining x and y as a function of c.

$$f\ (x, y, c(x,y))$$

Therefore from equation 3.1 and equation 3.7

$$\frac{\partial f(x,y,c)}{\partial c} = 2(x-c)(-\frac{\partial c}{\partial c}) + \frac{\partial y^2}{\partial c} = 0$$

$$= 2(x-c)(-1) = 0$$

or

$$x = c$$

Substituting this into equation 3.1 gives $y = \pm 1$. Thus the lines $y = +1$ and $y = -1$ are the envelopes of the family of circles.

The cam profile equations for different cam follower arrangements derived by using theory of envelopes are given below.

Disk cam with radially translating roller follower

$$X = R \cos\theta - \left(\frac{R_f}{(1 + (\frac{M}{N})^2)^{1/2}}\right)$$

$$Y = \frac{X M + R V}{N}$$

where $R = R_b + R_f + S$

R_b is the base circle radius

R_f is the roller radius

S is displacement of the follower

V is velocity of the follower

θ is cam rotation angle

$M = R \sin\theta - V \cos\theta$

$N = R \cos\theta + V \sin\theta$

Disk cam with offset translating roller follower

$$X = e \sin \theta + (d + S) \cos \theta - \left(\frac{R_f}{(1 + (\frac{U}{W})^2)^{1/2}}\right)$$

$$Y = \frac{X\,U + (d + S)\,V}{W}$$

where e is roller offset

$$d = ((R_b + R_f)^2 - e^2)^{1/2}$$

$$U = (d + S) \sin \theta - (V + e) \cos \theta$$

$$W = (d + S) \cos \theta + (V + e) \sin \theta$$

Disk cam with swinging roller follower

$$X = R_a \cos \theta - R_r \cos \delta + \left(\frac{R_f}{(1 + (\frac{P}{Q})^2)^{1/2}}\right)$$

$$Y = \frac{X\,P}{Q}$$

$$\delta = \theta - \phi - \cos^{-1} \frac{R_r^2 + R_a^2 - (R_b + R_f)^2}{2\,R_a\,R_r}$$

where R_a is distance between cam center and the follower center

R_r is the length of the roller follower arm

ϕ is displacement of the follower

$$P = R_a \sin \theta - R_r (1 - V) \sin \delta$$

$$Q = R_a \cos \theta - R_r (1 - V) \cos \delta$$

Disk cam with flat-faced translating follower

$$X = (R_b + S)\ \cos\ \theta\ -\ V\ \sin\ \theta$$

$$Y = (R_b + S)\ \sin\ \theta\ +\ V\ \cos\ \theta$$

Disk cam with centric flat-faced oscillating follower

$$X = R_a\ (\cos\ \theta + A\ \cos\ B)$$

$$Y = R_a\ (\sin\ \theta + A\ \sin\ B)$$

where $A = \frac{\cos\ (\theta +\ B)}{V-1}$

$B = \sin^{-1}\ (\frac{R_b}{R_a}) + \phi - \theta$

Disk cam with eccentric flat-faced oscillating follower

$$X = R_a\ (\cos\ \theta + A\ \cos B) + e\ \sin B$$

$$Y = R_a\ (\sin\ \theta + A\ \sin B) + e\ \cos B$$

where $A = \frac{\cos\ (\theta +\ B)}{V-1}$

$B = \sin^{-1}\ (\frac{R_b + e}{R_a}) + \phi - \theta$

3.2 Cutter Coordinates

For cam manufacturing, the location of milling cutter or of the grinding wheel i.e. the trace of the cutter or the grinder path should be known in order to produce the right cam profile. The cutter coordinate equations are not a simple variation of the profile equations, because the normal line at the point of

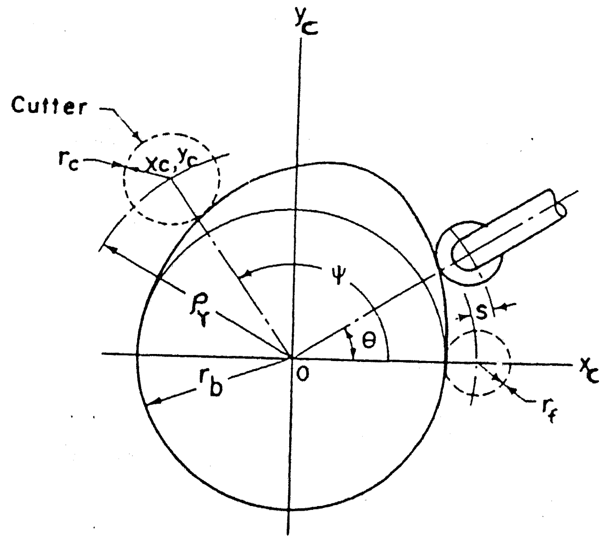


Figure 3.4: Cutter coordinates

tangency of the cutter and the profile does not continually pass through the the cam center.

If the milling cutter size is different from that of the follower , the geometry required to describe the cutter center is a circle representing the cutter and polar coordinates ρ_r and ψ locating the cutter center from cam center O as shown in fig. 3.4. The circle is tangent to the cam profile and has radius Rc. To locate the center in X-Y coordinate system the projection of ρ_r on the X-axis and on the Y-axis is first determined.

The mathematical expressions of Xc and Yc for various types of cam follower arrangements are given [1] below.

Disk cam with flat-faced translating follower

$$X_c = X + R_c \cos\theta$$

$$Y_c = Y + R_c \sin\theta$$

where X, Y are cam profile coordinates

Disk cam with radially translating roller follower

$$X_c = X + \left(\frac{R_c}{R_f} \right) (R \cos \theta - X)$$

$$Y_c = Y + \left(\frac{R_c}{R_f} \right) (R \sin \theta - Y)$$

Disk cam with offset translating roller follower

$$X_c = X + \left(\frac{R_c}{R_f} \right) (X_f - X)$$

$$Y_c = Y + \left(\frac{R_c}{R_f} \right) (Y_f - Y)$$

where $X_f = e \sin \theta + (d + S) \cos \theta$

$Y_f = -e \cos \theta + (d + S) \sin \theta$

Disk cam with oscillating roller follower

$$X_c = X + \left(\frac{R_c}{R_f} \right) (X_f' - X)$$

$$Y_c = Y + \left(\frac{R_c}{R_f} \right) (Y_f' - Y)$$

where $X_f' = R_a \cos \theta - R_r \cos \alpha$

$Y_f' = R_a \sin \theta - R_r \sin \alpha$

$\alpha = \theta - \phi - \phi_0$

Disk cam with centric flat-faced oscillating follower

$$X_c = X + R_c \cos\alpha$$

$$Y_c = Y + R_c \sin\alpha$$

where $\alpha = \phi - \theta + \phi_0$

Disk cam with eccentric flat-faced oscillating follower

$$X_c = X + R_c \cos\alpha$$

$$Y_c = Y + R_c \sin\alpha$$

where $\alpha = \phi - \theta + \phi_0$

Polar cutter coordinates can be obtained from

$$\rho_r = (X_c^2 + Y_c^2)^{1/2}$$

$$\psi = \tan^{-1}(\frac{Y_c}{X_c})$$

3.3 Pressure Angle

The pressure angle of a cam is the angle between the line of motion of the follower and the normal to the cam surface at the point of contact between the cam and the follower. This is represented by angle α in fig 3.5

The size of the pressure angle is important because

(1) Increasing the pressure angle increases the side thrust and this increases the forces exerted on cam and follower, resulting in jamming of the follower stem.

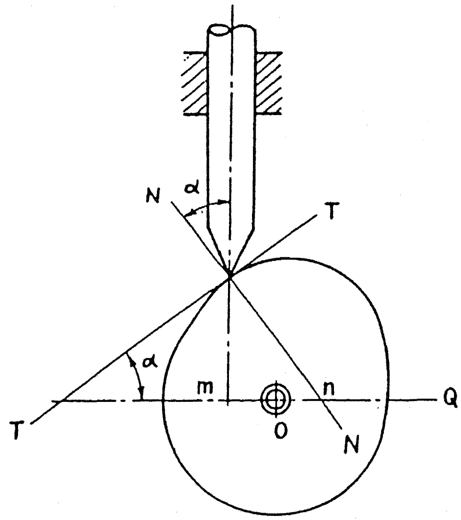


Figure 3.5: Pressure Angle

(2) Reducing the pressure angle increases the cam size and often this is not desirable, since there are practical limitations on the size of the cam.

The maximum pressure angle α_{max} should in general, be kept upto or below 30 degrees for translating type followers and upto or below 45 degrees for swinging type followers. In case of flat faced follower the pressure angle is negligible (almost zero). These values are on the conservative side and in many cases can be increased considerably, but beyond these limits trouble could develop and an analysis will be necessary. The maximum pressure angle establishes the cam size, torque, loads, accelerations, cam life and other pertinent factors.

Analytical expressions for calculations of pressure angle for different types of cam follower arrangements are given [8] below.

Disk cam with radially translating roller follower

$$\tan \alpha = \frac{\frac{ds}{d\theta}}{S + R_0} \tag{3.8}$$

where R_0 is prime circle radius

Disk cam with offset translating roller follower

$$\tan\alpha = \frac{\frac{ds}{d\theta} - e}{S + \sqrt{R_0^2 - e^2}} \tag{3.9}$$

Disk cam with oscillating roller follower

$$\tan\alpha = \cot\,(\phi + \phi_0) - \frac{R_r\,(1 - \frac{d\phi}{d\theta})}{R_a\,\sin\,(\phi + \phi_0)} \tag{3.10}$$

3.4 Cam Radius of Curvature

The radius of curvature of a curve is a measure of the rapidity with which the curve changes direction. The minimum radius of curvature of a cam should be kept as large as possible

- To prevent undercutting of the convex portion of the cam and
- To prevent very high surface stresses.

3.4.1 Curvature of a Curve

Suppose that a curve is given by the equation $y = f(x)$ and that f has a continuous second derivative. As shown in Fig 3.6, at a particular point $P_0(x_0, y_0)$ the tangent to the curve makes an angle γ with the positive x direction. From the definition of a derivative,

$$\gamma = f'(x_0)$$

or that at a point P(x, y)

$$\gamma(x) = tan^{-1}[f'(x)]$$

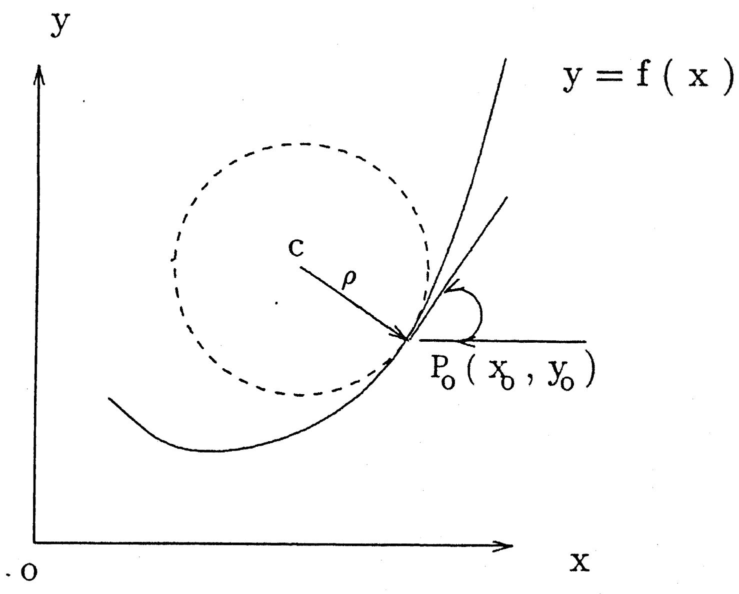


Figure 3.6: Curvature of a Curve

Therefore, the curvature k is the rate of change of the angle γ with respect to the arc length s.

$$k = \frac{d\gamma}{ds}$$

The radius of curvature ρ of an arc at a point is defined as the reciprocal the absolute value of the curvature at that point,that is

$$\rho = 1/|k|$$

The circle of curvature of an arc at a point P is that circle which passes through P that has a radius equal to ρ, and whose center c lies on the concave side of the of the curve along the normal through P.

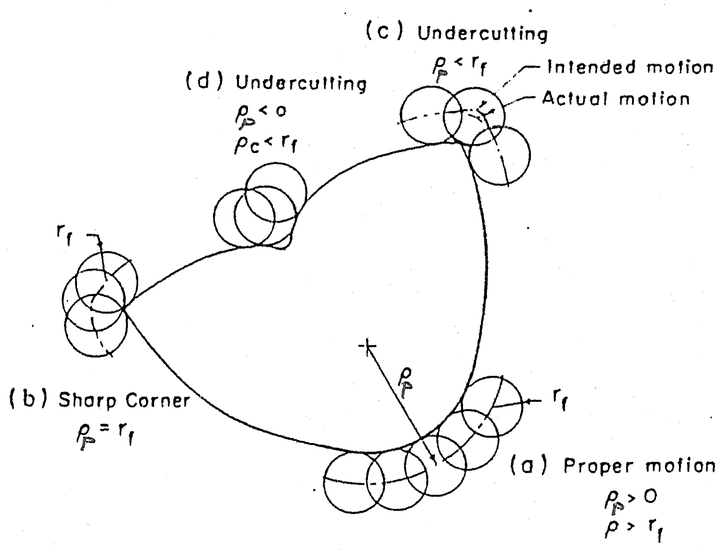


Figure 3.7: Undercutting of cam with roller follower

3.4.2 Undercutting Phenomenon

When the follower or cam cutter can not follow the desired cam path, a condition known as "undercutting" will result.

Undercutting of Cam with Roller Follower

With a roller follower, undercutting occurs in a convex curve when the radius of curvature of the pitch curve ρ_p is less than the radius of the roller R_f. With a concave curve, it occurs when the radius of curvature of the cam profile ρ_c is less than R_f.

Fig. 3.7 illustrates the undercutting conditions of a cam with a roller follower. Point (a) in figure 3.7 shows a proper operating condition without undercutting. The radius of curvature of the convex part of the follower is ρ_p, and cam will at that point have a cam curve radius of curvature of $\rho_c = \rho_p - R_f$.

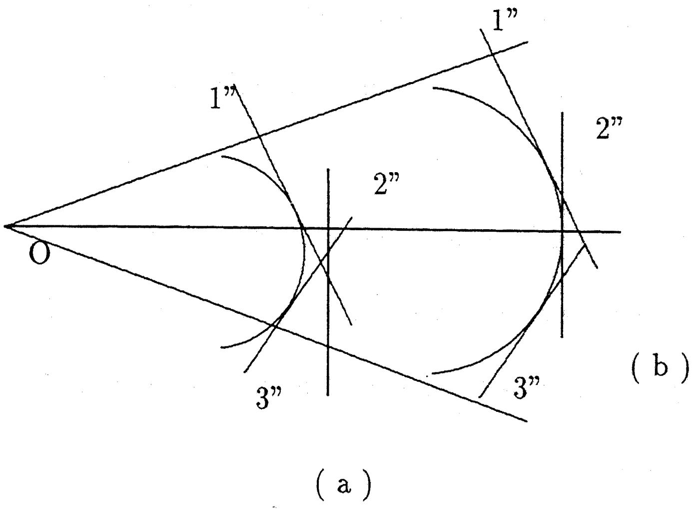


Figure 3.8: Undercutting of cam with flat-faced follower

At point (b) of in fig. 3.7, the conditions $\rho_p = R_f$ and $\rho_c = 0$ arises with the convex curve. Therefore, the actual cam will have a sharp corner. Theoretically, the stresses at this sharp corner are infinite in magnitude, and the cam surface wears rapidly. At point (c) in figure 3.7 it is shown again with a convex curve for the case where $\rho_p < R_f$. This case is not possible because undercutting will result, and the actual motion of the follower will deviate from the intended one.

Undercutting is not likely to occur on the concave portion of the cam curve, but the case should be exercised in ensuring that the radius of curvature of the actual cam curve is not equal to or less than the radius of roller. This condition occurs if there is a concavity on the displacement curve, such as the one shown at point (d) in fig 3.7.

Undercutting of Cam with Flat-faced Follower

Fig 3.8 (a) shows the face of a flat-faced follower in its relative positions $1''$,

2'', and 3'', and the cam drawn tangent to these lines. The cam profile drawn tangent to the flat-faced follower, cannot reach tangency to the face at position 2''. In this situation the follower positions 1'' and 3'' eliminate or undercut position 2''. The cam thus developed is incapable of driving the follower according to the originally intended movement. Solution to this problem is to increase the cam size until the undercutting phenomenon disappears. This is depicted in fig 3.8 (b).

To avoid undercutting of cam with flat-faced translating follower, ρ_p should be greater than zero.

The relations to find radius of curvature of cam are given [8] below

Disk cam with radially translating roller follower

$$\rho_p = \frac{[(R_0 + S)^2 + (S')^2]^{3/2}}{(R_0 + S)^2 + 2 (S')^2 - (R_0 + S) S''} \tag{3.11}$$

Disk cam with offset translating roller follower

$$\frac{1}{\rho_p} = \Gamma[1 + \Gamma(S' \sin\alpha - S'' \cos\alpha) \tag{3.12}$$

where . $\Gamma = \frac{\cos\alpha}{S+\sqrt{R_0^2 - e^2}}$

$S' = \frac{ds}{d\theta}$

$S'' = \frac{d^2 s}{d\theta^2}$

Disk cam with oscillating roller follower

$$\frac{1}{\rho_p} = \gamma[1 + [(1 - \phi') \phi' \sin\alpha - \phi'' \cos\alpha]] \tag{3.13}$$

where . $\gamma = \frac{\cos\alpha}{R_a \sin \delta}$

$\delta = \phi + \phi_0$

$\phi' = \frac{d\phi}{d\theta}$

$\phi'' = \frac{d^2\phi}{d\theta^2}$

Disk cam with flat-faced translating follower

$$\rho_p = S + R_b + A \tag{3.14}$$

Disk cam with oscilating flat-faced follower

$$\rho_p = \gamma[B + \ (\ (1 - 2\phi') \ sin(\delta)] - e \tag{3.15}$$

where . $\gamma = \frac{R_a}{(1-\phi')^2}$

$B = \phi'' \frac{\cos\delta}{(1-\phi')}$

$\delta = \phi + \phi_0$

Positive ρ_p refers to a convex pitch profile; negative ρ_p refers to concave pitch profile. The radius of curvature of working profile is $\rho_p - R_f$ for a convex profile and $\rho_p + R_f$ for a concave one.

The radius of curvature decreases with decrease in the prime circle radius of the roller follower. The radius of curvature must be greater than roller radius. A roller size that is as small as possible should be used.

To minimize wear, the radius of curvature should be as large as possible, but it generally has a practical limitations.

Other cam dimensional parameters (such as the amount of offset in cams with reciprocating followers and pivot location and follower length with swinging followers) will also affect the radius of curvature, but their effects are not critical.

3.5 Contact Stresses

The main factors influencing the cam forces are :

1. Displacement and cam speed (forces due to acceleration)
2. Dynamic forces due to backslash and flexibility
3. Linkage dimensions which weight and weight distribution
4. Pressure angle and friction forces
5. spring forces

The main factors influencing stresses in cams are :

- Radius of curvature for cam and follower
- Materials

When two elastic bodies are in contact, the compressive stress developed at the surface of contact is calculated from Hertz's stress model [3].

Hertz's model : when two cylinders of different materials are in contact.

$$\sigma = 0.564 \left[\frac{\frac{F}{W}\left(\frac{\rho_c+\rho_f}{\rho_c\,\rho_f}\right)}{\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}} \right]^{1/2} \qquad (3.16)$$

where . σ is contact stress between cam and follower

F is total external load on the follower, including static weight, spring force, inertia etc.

W is width of the follower

ρ_c, ρ_f are radius of curvature of cam & follower resp.

μ_1, μ_2 are poissons ratio of cam & the follower resp.

E_1, E_2 are modulus of elasticity of cam & the follower resp.

Hertz's model : when two cylinders of same materials are in contact.

$$\sigma = 0.418 \; [\frac{F\,E}{W} \; (\frac{\rho_c + \rho_f}{\rho_c \; \rho_f})]^{1/2} \tag{3.17}$$

Equations 3.16 and 3.17 are useful only for cams with roller followers.

Hertz's model : when cylinder is in contact with flat surface of similar metal.

$$\sigma = 0.564 \; [\frac{\frac{F}{W} \; (\frac{\rho_c+\rho_f}{\rho_c \; \rho_f})}{\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}}]^{1/2} \tag{3.18}$$

Hertz's model : when cylinder is in contact with flat surface of different metal.

$$\sigma = 0.418 \; [\frac{F\,E}{W\,\rho}]^{1/2} \tag{3.19}$$

Equations 3.18 and 3.19 are useful only for cams with flat-faced followers.

3.5.1 Choice of Cam Materials

In considering materials for cams it is difficult to select any single material as being the best for a practical application. Often the choice is based on custom, and on the machinability of the materials rather than its strength. However,

Table 3.1: Some proven metal combinations [3]

Cast iron and phosphor bronze
Hardened steel and phosphor bronze
Cast iron and soft steel
Babbit and soft steel
soft brasses and soft steel
Hardened steel and soft bronze
Hardened steel and cast iron
Hardened steel and brass
Hardened steel and nylon

the failure of a cam or follower is commonly due to fatigue, so that an important factor to be considered is the limiting wear load which depends upon the surface endurance limits of the materials used and relative hardness of the mating surfaces. In table 3.1 some proven metal combinations of cam and follower are given. Table 3.2 shows maximum permissible compressive stresses for various cam and follower material combinations.If the Hertz's compressive stress σ is kept below the values given in the table, the life of contacting surfaces will be more than 300 x 10^6 cycles.

3.6 Optimum Cam Size

Smooth and noiseless motion is an important objective of cam design. To achieve this the designer must create a minimum-sized cam

- To limit the pressure angle to prevent jamming between the follower stem and its guide, and to obtain as little side thrust against the follower as

Table 3.2: Choice of materials for cams and followers. [3]

Follower surface material	Cam surface material	Allowable compressive stress (N/mm^2)	Code
Hardened and ground steel, RC 51-52	Gray cast iron	389.00	A
	Nickel cast iron - as cast	455.00	B
	Nickel cast iron - heat treated, RC 35-40	545.00	C
	Molybdenum cast iron - as cast	565.00	D
	Molybdenum cast iron - hot quench treatment	780.00	E
1.05 % carbon tool steel RC 60-63	Low carbon, free machine steel, case hardened, RC 56-61	1720.00	F
	SAE 4340, induction hardened, RC 48-50	1720.00	G
Hardened tool steel	Medium carbon low alloy (0.47% c, 1% Mn, 0.65% Cr, 0.2% Mo) machine steel, RC - 30	1585.00	H
Hardened & ground steel, RC 51-52	SAE 1010 - induction hardened, RC - 60	1720.00	I
	SAE 2515 - ground & lapped, case hardened	1720.00	J
Nickel cast iron hot quench	Phosphor bronze, SAE 65	570.00	K
Hardened & ground steel, RC 51-52	Phosphor bronze - SAE 65, 80 BHN	565.00	L
	Nickel bronze - 80 BHN	525.00	M

possible.

- To limit the minimum value of cam profile radius of curvature to avoid undercutting, which results in incorrect follower movement and cause large contact stresses between the cam and the follower.

Also, bigger cam size is not desirable because

- The size of the cam determines, to a certain extent, the size of the machine.
- Large cams require more precise cutting points in manufacturing and therefore an increase in cost.
- Large cams mean that the circumferential speed of the cam is high and small deviation from the theoretical path of the follower cause additional acceleration, the size of which increases with the square of the size.
- Large cams mean more revolving weight and in high speed machines this leads to increased vibrations in the machine.
- The inertia of a large cam may interfere with quick starting and stopping.

The problem is solved by selecting the proper follower offset for the smallest possible cam. The optimum cam size and follower offset are basically affected by (1) the maximum permissible pressure angle or steepness of its profile, (2) the cam radius of curvature or sharpness of the profile and (3) the size of the cam shaft. In general, the pressure angle decreases and radius of curvature increases when the base circle radius of cam increases.

Radial translating roller follower

Differentiating equation 3.8 with respect to θ and setting it to zero and simplifying we get,

$$R_p = \frac{\left(\frac{ds}{d\theta}\right)_p^2}{\left(\frac{d^2s}{d\theta^2}\right)_p} \tag{3.20}$$

where $R_p = R_0 + S$

R_p is the radius of the pitch curve in contact with the reference point of the follower.

θ_p is the cam angle in radians through which the cam turns while the follower rises from the rest to where the pressure angle is maximum.

where the subscript p refers to the pitch point value.

from equation 3.8

$$\tan \alpha_m = \frac{\left(\frac{d^2s}{d\theta^2}\right)_p}{\left(\frac{ds}{d\theta}\right)_p} \tag{3.21}$$

where α_m = the maximum pressure angle

solving equation 3.20 and equation 3.21 gives the smallest cam with a prescribed pressure angle.

In equation 3.20 and equation 3.21 to find θ_p at α_{max} so as to find velocity and acceleration at α_{max} is difficult task. It involves the solution of complicated equations. The closed form solution exists only for some basic motion curves (SHM, Parabolic). This can normally be accompished by a mathematical search technique for the maximum through computer programming.

In case of the flat-faced follower the pressure angle is zero. So minimum cam size is based on the radius of curvature of cam. An analytical method [2] can

be used to determine the length of the follower face and minimum radius of cam to avoid undercutting or cusps.

The length l of the follower face is given by the relation

$l \ = \ S'(\theta)$

and to avoid cusps following relation should be satisfied.

$$R_b + S(\theta) \ + \ S''(\theta) \ > \ 0 \tag{3.22}$$

The sum $[S(\theta) \ \ + S''(\theta) \ ]$ must be inspected for all values of θ to determine its minimum algebraic value. It is necessary to use the minimum value of the sum so that R_b will be sufficiently large to insure that equation 3.22 does not became zero for any value of θ.

The size of the cam can be diminished further. An obvious solution is either to extend the time for rise and/or return; to choose another displacement diagram having a lower maximum, or to decrease lift by increasing the lever ratio of the linkages. This sort of investigation is often necessary and requires that one know the functioning of the machine very well.

Chapter 4

Implementation

Implementation of present work has been done in 'C' in UNIX environment on HP work station. The 'C' language was chosen mainly because of its association with UNIX and the dynamic allocation facility thereby costing only the required amount in terms of memory. Besides these, more efficiency during execution as compared to other higher level languages is achieved due to compact code generated by C compiler. The software is built with the basic idea of designing a mechanical system in front of terminal, with sufficient flexibility given to the user to change various parameters and observe the results. The displacement, velocity and acceleration diagrams, cam profile and cutter path can be seen on graphic output. This chapter describes the salient features of implementation of the system.

4.1 User Interface

In cam design problems, certain prescribed conditions must be fulfilled for a design to be acceptable, and more than one solution usually satisfies the requirements of a particular application. A straightforward procedure may not be available, and an optimum design is only obtained after many trials, each with slightly different input data. The cam design with graphics seeks to improve this by providing a fast man-machine interaction, whereby the designer may change the design, have the results displayed on the display screen and manipulate the input parameters in various combinations until an acceptable cam profile is obtained. By showing the result in graphic form, the operator can quickly examine the overall characteristics of the resultant system and take immediate action. Errors are detected, and time loss because of erroneous data is effectively minimized. Program checks and warning messages are available to guide the user during the design operations and to indicate invalid conditions. The system is designed to be totally interactive and user friendly..

Strategy of System Design

The system is based on the strategy of modular design. The system consists of various modules. The different modules are

- Data input module
- Synthesis module
- Cam-cutter module

- Analysis module
- Optimization module
- Manufacturing module

The modular programming facility gives the scope to creat different modules in the program. These modules are enable to exchange data information through data files.

4.2 Data input module

In this module the system asks the user to enter the various input parameters such as, totallift of the follower, total cam rotation angle, types of displacement curve and no. of motion curves.

The task of the designer in selecting the motion curves is reduced considerably. A submodule 'Help' is incorporated for this purpose. Since system handles wide variety of motion curves, the designer should know the characteristics of each motion curve and their C_v and C_a values. In help module this task is simplified. The system displays table 2.2 and table 4.1 on the screen. In table 2.2 types of displacement curve, required boundary conditions and recommended types of motion curve for each type of displacement curve is presented. Table 4.1 provides information related to velocity factor C_v acceleration factor C_a and speed application of each motion curve. The curves are coded as shown in the table 4.1. The module also provides the information about how to use these tables.

Table 4.1: Motion curves : velocity factor, acceleration factor

Motion Curves		Velocity	Acceleration	digit
Rise	Code	Factor	Factor	Vector
Half Cycloidal (accl.)	CM_1	2.0	π	1121
Half Cycloidal (deccl.)	CM_2	2.0	π	2111
Cycloidal	CM_3	2.0	2π	1111
Half Harmonic (accl.)	SH_1	$\pi/2$	2.4560	1221
Half Harmonic (deccl.)	SH_2	$\pi/2$	2.4560	2113
Simple Harmonic	SH_3	$\pi/2$	4.9300	1213
Half 3-4-5 Poly. (accl.)	5P_1	1.890	2.8850	1121
Half 3-4-5 Poly. (deccl.)	5P_2	1.890	2.8850	2111
3-4-5 Polynomial	5P_3	1.890	5.7700	1111
Half 7^{th} Poly. (accl)	7P_1	2.190	3.7600	1121
Half 7^{th} Poly. (deccl)	7P_2	2.190	3.7600	2111
7^{th} degree Poly.	7P_3	2.190	7.5200	1111
Modi. Trapezoidal	MT_1	2.0	4.8900	1111
Modified sin	MS_1	1.76	5.5300	1111
5^{th} degree Poly.	5P_7	1.5625	6.6667	1113
8^{th} degree Poly.	8P_1	—	5.2683	1113
Return	Coding			
Half Cycloidal (accl.)	CM_4	2.0	π	1131
Half Cycloidal (deccl.)	CM_5	2.0	π	3111
Cycloidal	CM_6	2.0	2π	1111
Half Harmonic (accl.)	SH_4	$\pi/2$	2.4560	1331
Half Harmonic (deccl.)	SH_5	$\pi/2$	2.4560	3112
Simple Harmonic	SH_6	$\pi/2$	4.9300	1312
Half 3-4-5 Poly. (accl.)	5P_4	1.890	2.8850	1131
Half 3-4-5 Poly. (deccl.)	5P_5	1.890	2.8850	3111
3-4-5 Polynomial	5P_6	1.890	5.7700	1111
7^{th} Poly. (accl.)	7P_4	2.190	3.7600	1131
7^{th} Poly. (deccl.)	7P_5	2.190	3.7600	3111
7^{th} degree Polynomial	7P_6	2.190	7.5200	1111
Modi. Trapezoidal	MT_2	2.0	4.8900	1111
Modified Sine	MS_2	1.76	5.5300	1111
5^{th} degree Poly.	5P_8	1.5625	6.6667	1311
8^{th} degree Poly.	8P_2	—	5.2683	1311

Table 4.2: Digit vector representation

	Digit component		
Order of digit	1	2	3
First	$V_i = 0$	$V_i > 0$	$V_i < 0$
Second	$A_i = 0$	$A_i > 0$	$A_i < 0$
Third	$V_f = 0$	$V_f > 0$	$V_f < 0$
Fourth	$A_f = 0$	$A_f > 0$	$A_f < 0$

Table 4.1 also shows digit vectors obtained from boundary conditions of respective curves (see Table 4.2) to help the user in selecting the motion curves, when two or more curves are to be combined. To match any two curves, last two digits of digit vector of the first curve and first two digits of digit vector of the second curve should be same.

Fig. 4.1 shows various steps of the Data input module.

4.3 Synthesis Module

Based on the fundamental conditions of combining cam curves the lift and rotation angle of each segment can be calculated. The system will ask one or more additional input if current input is insufficient to design a cam. The fundamental conditions and motion curves used for synthesis are described in the Chapter 1.

Once the lift and rotation angle of each curve is available , displacement, velocity and acceleration of each segment is calculated. The plot of these parameters versus cam rotation angle is displayed on the screen. The system at this stage

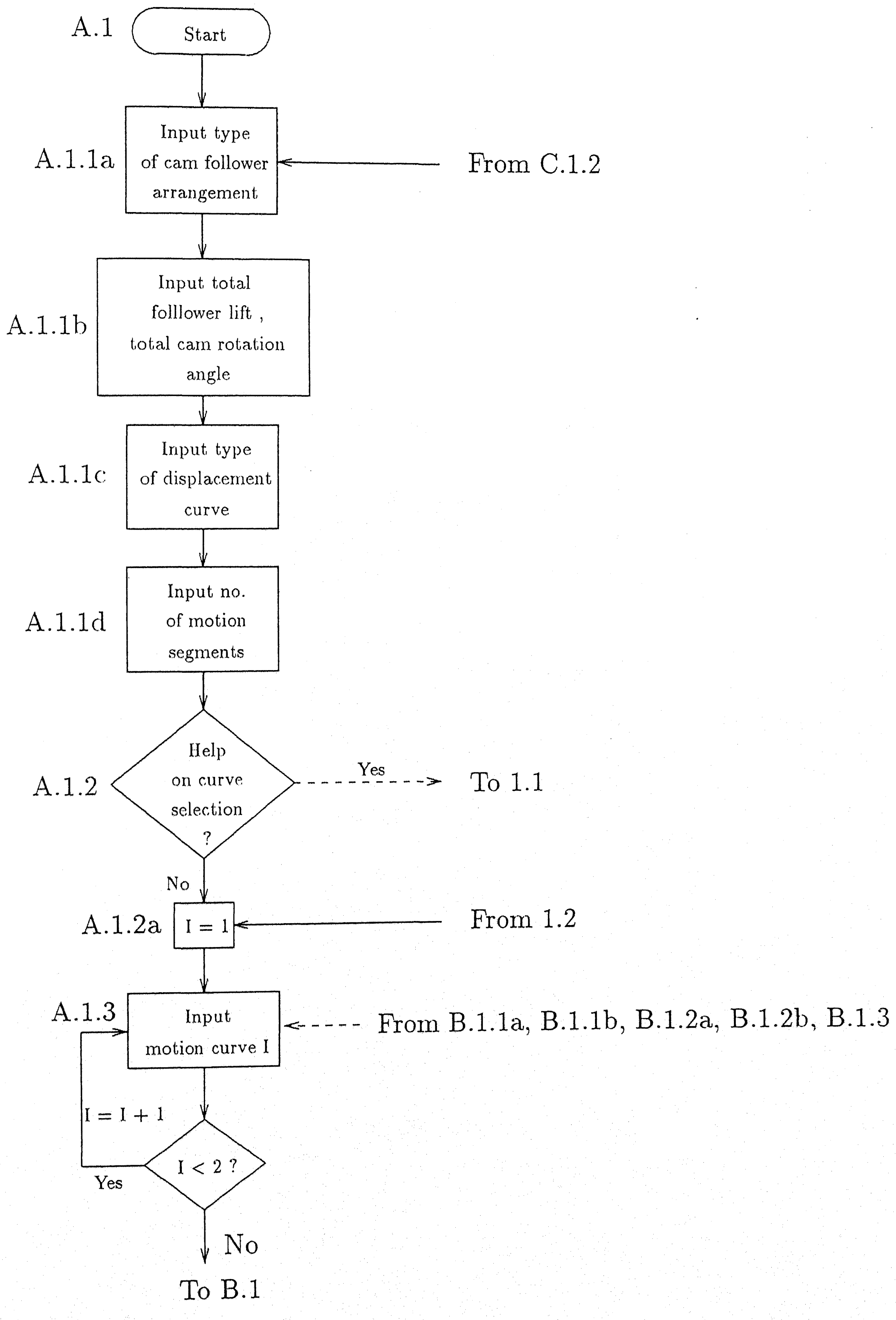


Figure 4.1: Various steps of Data input module

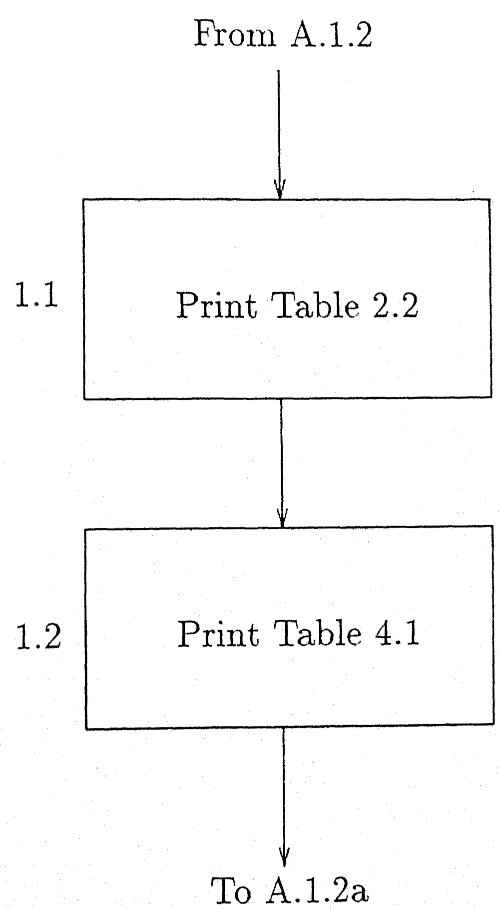


Figure 4.2: Help module

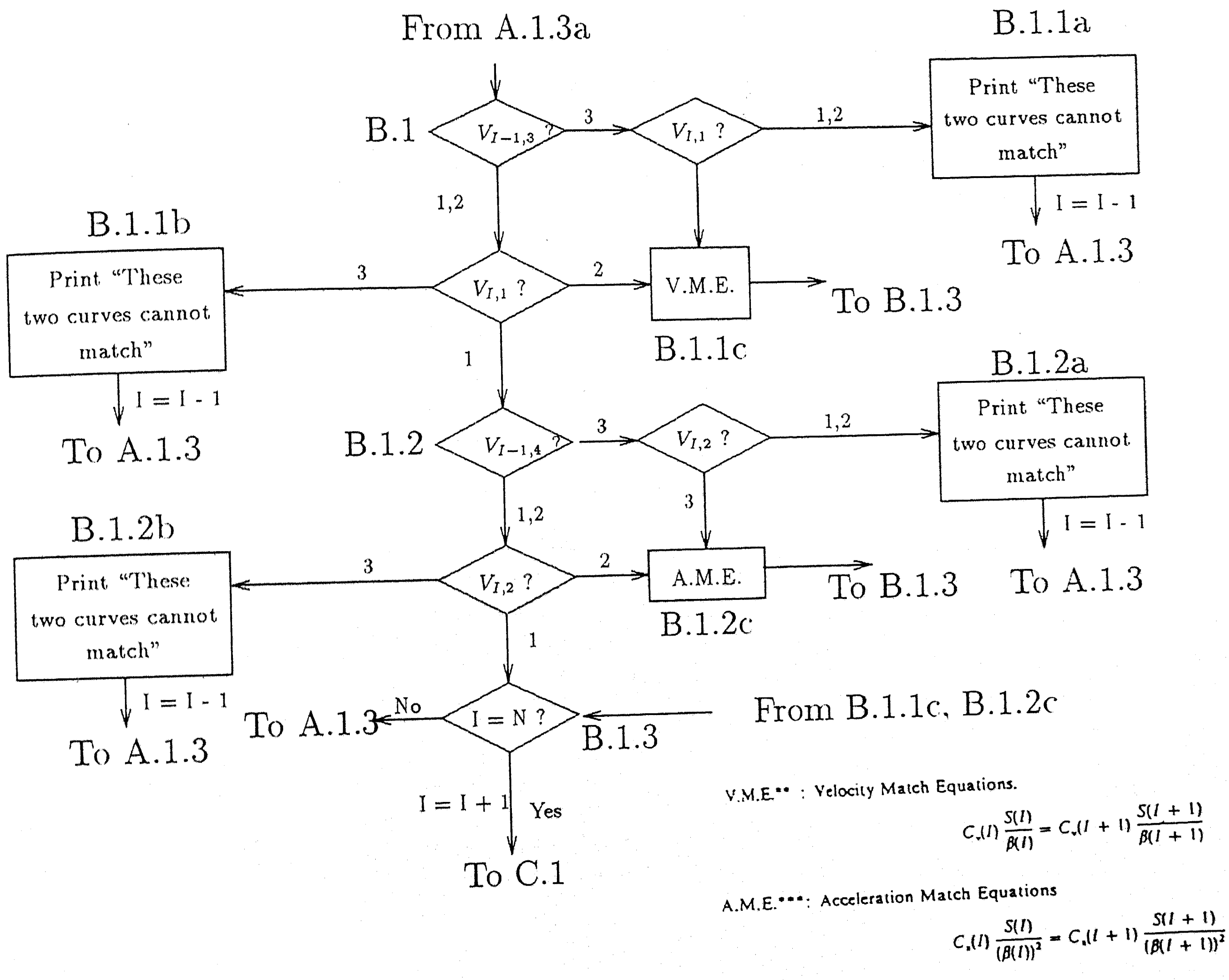


Figure 4.3: Various steps of synthesis module

provides option to change the input parameters. If these plots are acceptable user can proceed to next module, otherwise he can return to the input module to change the parameters. The values of displacement, velocity and acceleration are stored in the data files. Fig. 4.2 shows Help module.

Fig. 4.3 and Fig 4.4 shows various steps of the Synthesis module.

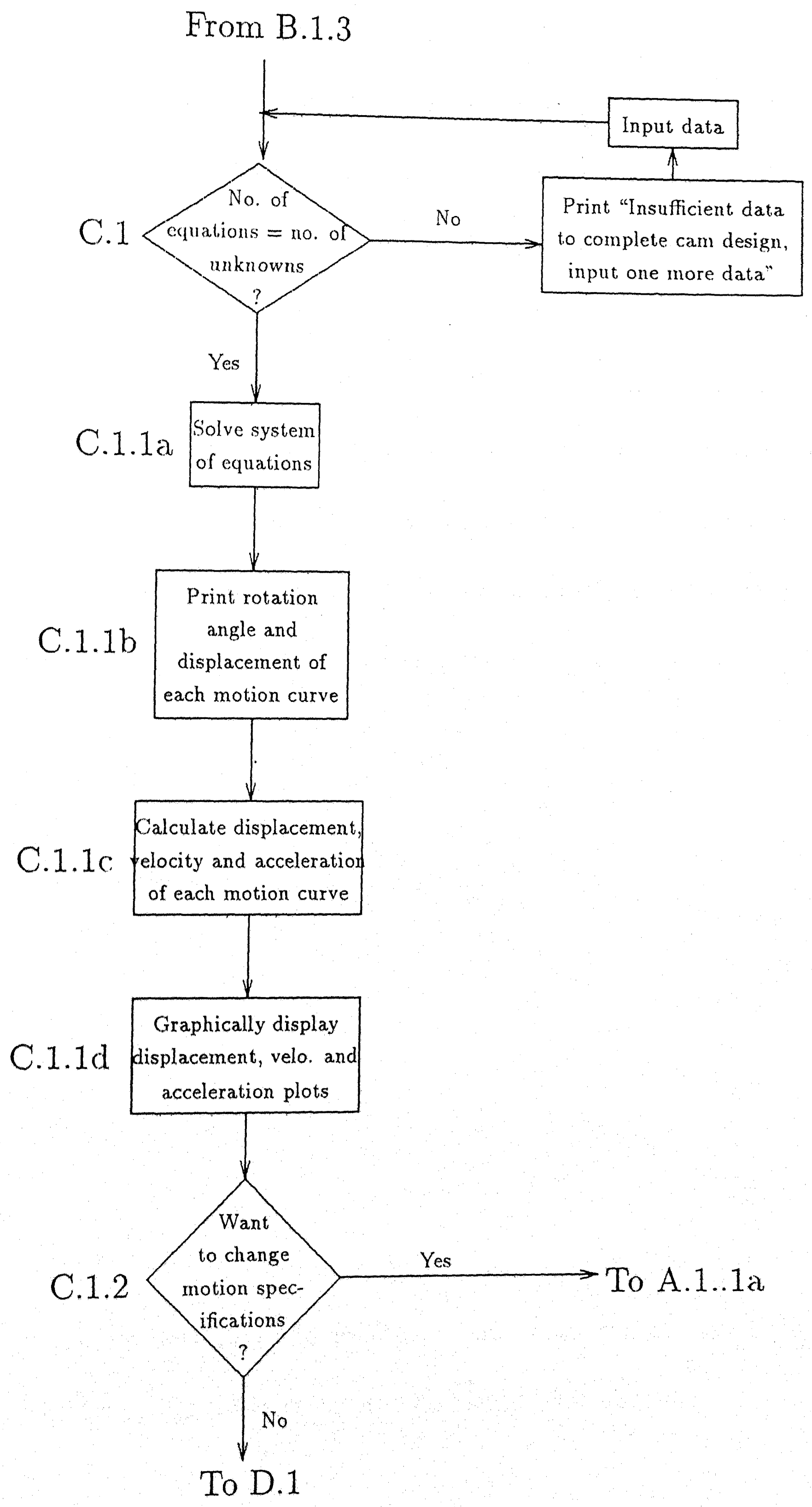


Figure 3.4: Synthesis module continue

4.4 Cam-Cutter Module

After the synthesis is over, next step is to obtain the cam profile. This module is concerned with the calculation of cam profile coordinates and cutter location data. The cam design is based on the envelope theory described in the Chapter 3. The user has to specify the parameters such as follower offset, base circle radius, roller diameter, follower pivot to cam center distance. The user also has to specify the direction of rotation of cam, cutter diameter and the coordinate system in which user wants the cutter location data. The cam profile coordinates and cutter location data is calculated for small increments of cam rotation angle and are stored in the files. The system displays cam profile and cutter path for visual inspection. It also displays the cutter at its incremental position to verify the tool path.

Block diagram of this module is shown in fig. 4.5.

4.5 Analysis Module

Only cam profile and cutter data is not sufficient, the designed cam should be analyzed for basic constraints. The constraints and their interrelations are described in Chapter 3. These constraints are pressure angle, radius of curvature and contact stress between cam and follower. This module consists of three submodules Pressure module, Curvature module and Stress module.

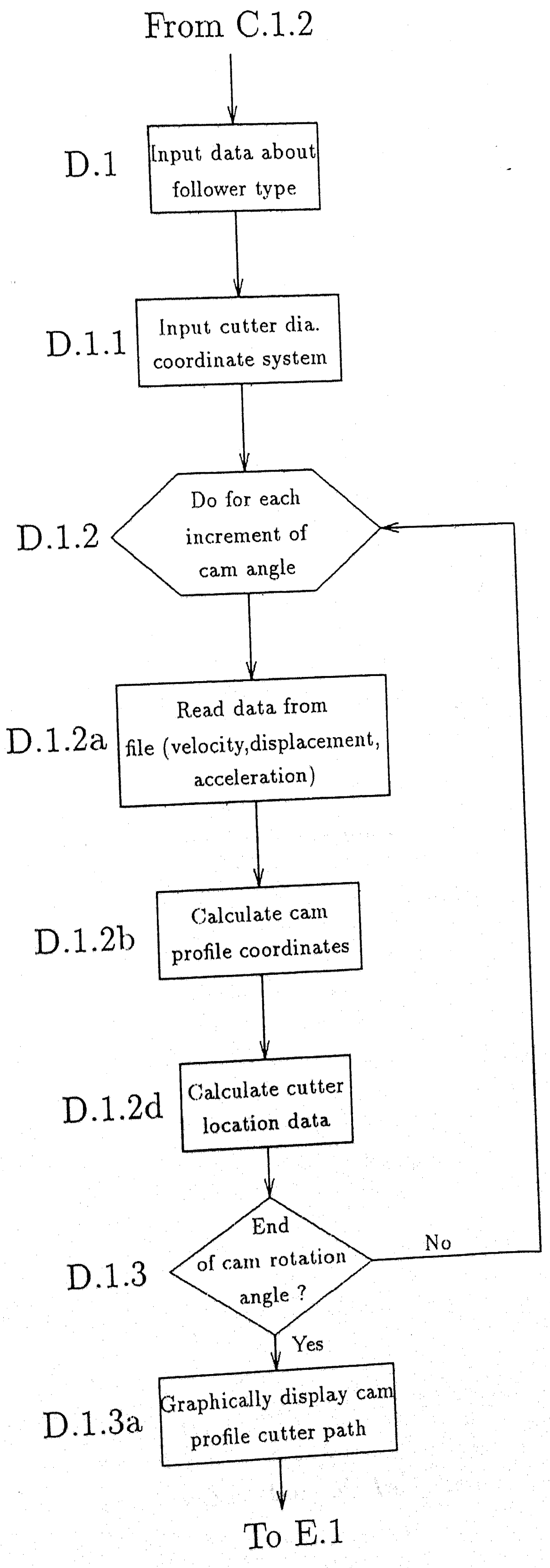


Figure 4.5: Block diagram : Cam-cutter Module

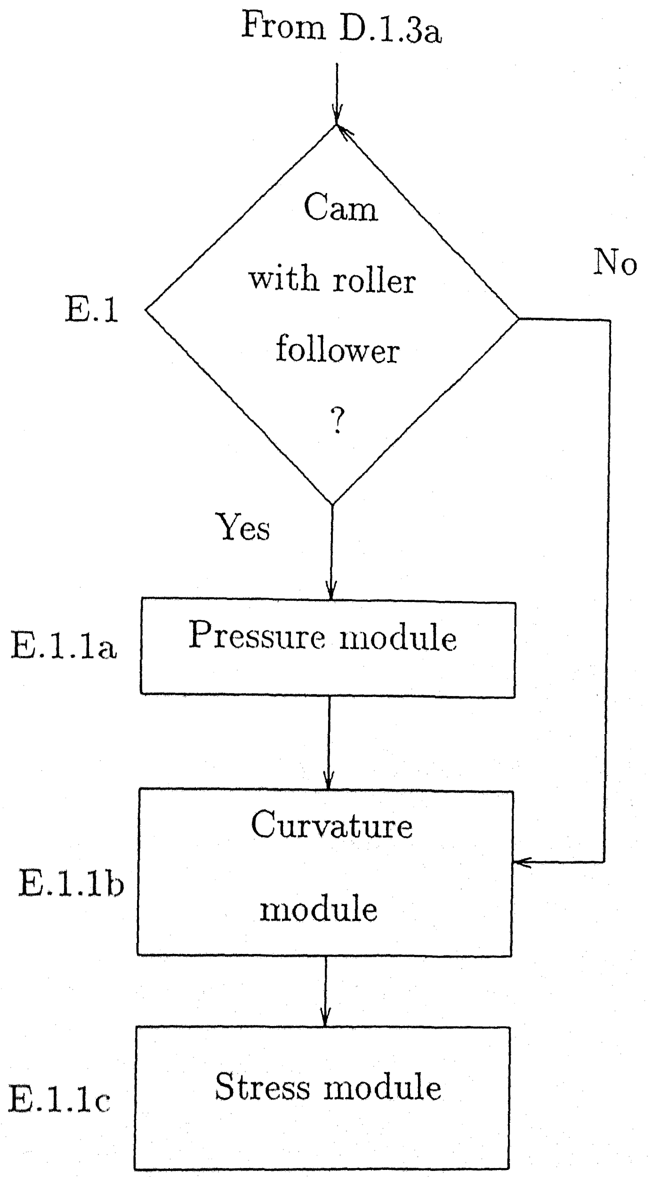


Figure 4.6: Various steps of Analysis module

Pressure Module

The designed cam should be checked for maximum pressure angle in order to prevent jamming of the follower stem. This module calculates pressure and keeps track of maximum pressure angle during rise and return of the follower. It warns the user if pressure angle at a particular point exceeds the maximum permissible value. It also specifies the location of maximum pressure angle in degrees of cam rotational angle.

fig. 4.7 shows block diagram of Pressure module.

Curvature Module

When follower does not follow the desired path then undercutting of cam takes place. The minimum radius of curvature is the factor which checks whether there is any undercutting. The Curvature module calculates radius of curvature of cam profile, minimum radius of curvature. It displays appropriate message to warn the user. This module also checks whether cutter diameter is small enough to manufacture the cam.

fig. 4.8 shows block diagram module.

Stress Module

Generally cam fails by fatigue, the selection of proper cam and follower material combination is restricted by the contact stress between cam and follower. The contact stress should not exceed the maximum permissible compressive stress.

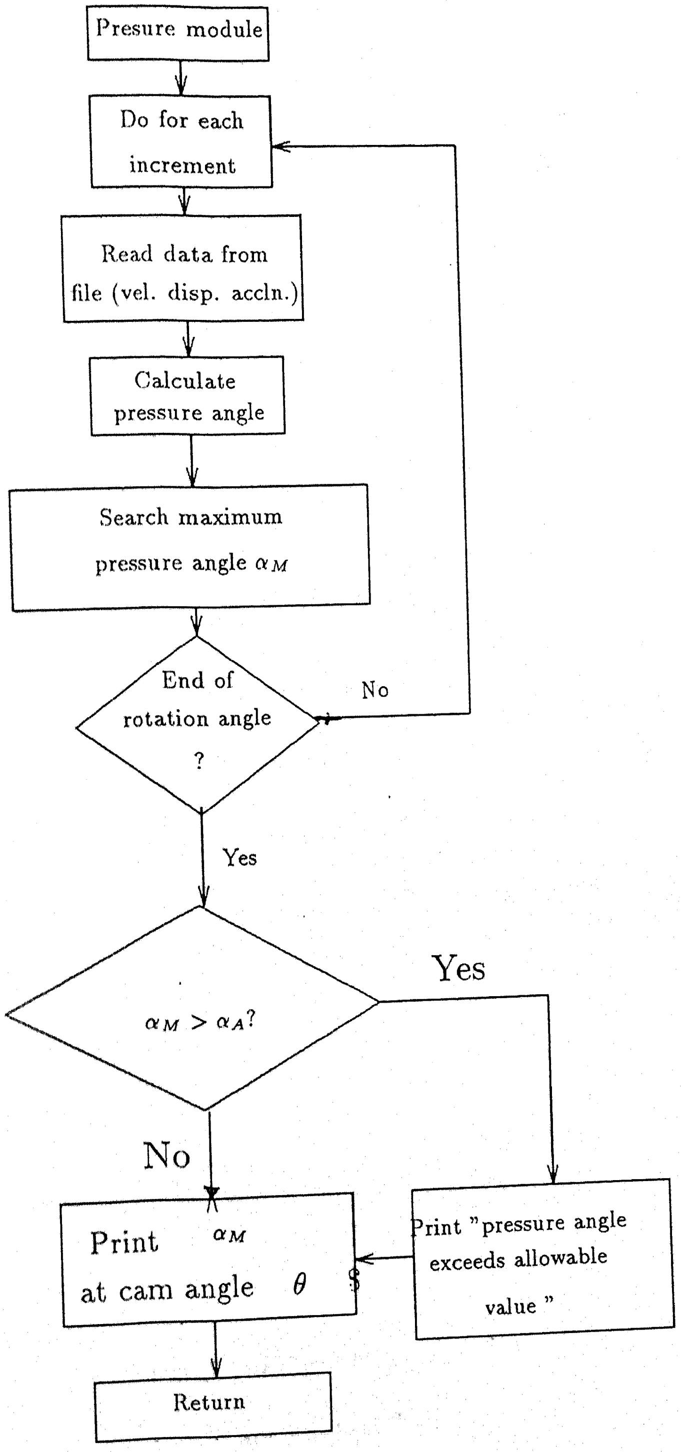


Figure 4.7: Block diagram : Pressure Module

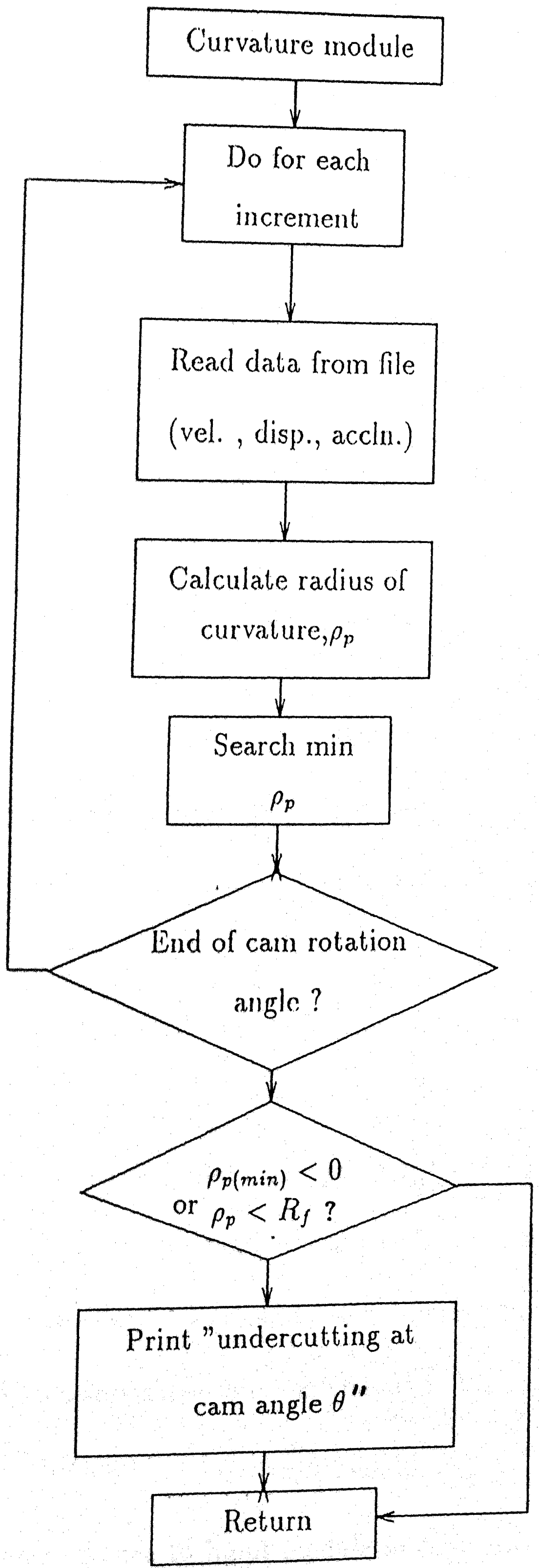


Fig 4.8 Block diagram : Curvature Module

The user has to select the cam and follower material combination from the list displayed by the system. The Stress module calculates contact stress from Hertz's model. It keeps track of maximum contact stress during rise and return of the follower. It also specifies the location of maximum contact stress in degrees of cam rotation angle. It checks the maximum contact stress with the maximum permissible compressive stress for particular metal combination from the database and displays appropriate message at the terminal.

Table 3.1 shows some proven combinations of materials for cam and follower.

4.6 Optimization Module

The objective of this module is to obtain minimum base circle radius. Also after analysis, if cam is not satisfying the basic constraints, user can return to Cam_cutter module to change the parameters which affects the pressure angle and radius of curvature of cam. It will be better if user knows the optimum parameters satisfying the basic constraints for the prescribed motion specification. The Optimization module suggest optimum values of cam base circle radius, follower offset (translating roller follower), pivot to cam center distance (swinging roller foller) and length of flat-face (translating flat face follower), for prescribed motion specifications and maximum pressure angle (roller followers). For flat faced followers these values are based on the radius of curvature of cam to avoid undercutting.

The user can also return to Input module to change the motion specifications,

since size of the cam can be diminished further by either extending the time for rise and or/return; by choosing another displacement curve having a lower maximum velocity.

4.7 Manufacturing Module

This module is concerned with the NC code generation for manufacturing of designed cam. The cutter location data calculated by the Cam_cutter module is input to this module. Once cutter location data in small increments of cam rotation angle is available, the system automatically generates the NC code. Naturally, the smaller the increments of cam angle, the better the cam finish will be. The increments in cam angle is restricted by the resolution of the machine on which cam is to be manufactured.

Chapter 5

Results and Discussion

5.1 Results

The working of the system can very well be explained by implementing the system to the following examples. The step by step interactive dialogues between the user and the system is shown in the trial run of example - 1.

5.1.1 Example - 1

Fig. 5.1 shows a typical DRRD displacement diagram of cycloidal-harmonic curve. The curve segments used are

AB - Cycloidal, rise

BC - Harmonic, rise

CD - Harmonic, return

DE - Cycloidal, return

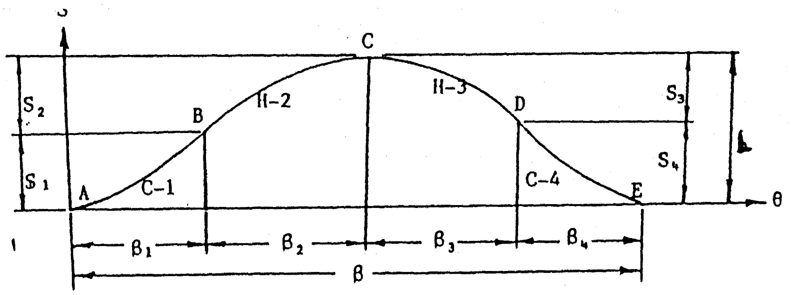


Figure 5.1: Displacement diagram of example - 1

Let the total displacement of the follower be 35. mm and total cam rotation angle 260°. It is given that $S_1 = S_2$ and $S_1 = 20$ mm. The cam is to be operated with radially translating roller follower. The width of the follower roller is 15 mm. The cam and the follower are made of cast iron and steel respectively. The permissible pressure angle is 30° for both rise and return. The task is to design an optimum cam for given motion specifications and prescribed pressure angle. The total load on the follower is 2900 N. Let the base circle radius of cam be 50. mm, follower roller radius 10 mm and cutter radius 15. mm.

Trial Run of Example - 1

When the user starts the program, system asks for various inputs interactively.

(1) In the first step, the list of various cam follower mechanisms will appear on the screen, and the system will ask the user to enter the appropriate code.

TYPES OF CAM FOLLOWER CONFIGURATIONS	CODES
FLAT-FACED TRANSLATING FOLLOWER	– 1
RADIALLY TRANSLATING ROLLER FOLLOWER	– 2
OFFSET TRANSLATING ROLLER FOLLOWER	– 3
SWINGING ROLLER FOLLOWER	– 4
CENTRIC SWINGING FLAT-FACED FOLLOWER	– 5
ECCENTRIC SWINGING FLAT-FACED FOLLOWER	– 6

ENTER THE FOLLOWER TYPE : *2*

(2) In the second step input the motion specifications.

ENTER TOTAL FOLLOWER DISPLACEMENT (mm) : *35.*

ENTER TOTAL CAM ROTATION ANGLE (degree) : *260.*

ENTER NO. OF CURVE SEGMENTS : *4*

(3) Now the system asks the user whether he wants help on curve selection or not.

DO YOU WANT HELP ON CURVE SELECTION (Y/N) ? : *Y*

If user enters yes, Table 2.2 and Table 4.1 will appear on the screen. From these tables user will get help in selecting the motion curves.

(4) In the next step, the system asks the user to input the motion curves. If user inputs wrong motion curves, the system will display appropriate message.

ENTER THE MOTION CURVE FOR SEGMENT NO. 1 : *CM_1*

ENTER THE MOTION CURVE FOR SEGMENT NO. 2 : *SH_1*

THESE TWO CURVES CAN NOT MATCH, TRY AGAIN

ENTER THE MOTION CURVE FOR SEGMENT NO. 2 : *SH_2*

ENTER THE MOTION CURVE FOR SEGMENT NO. 3 : *SH_4*

ENTER THE MOTION CURVE FOR SEGMENT NO. 4 : *CM_5*

(5) If the input data is insufficient to design a cam, the system will ask for one or more additional conditions.

ENTER DISPLACEMENT FOR SEGMENT NO. 1 (mm) : *20.*

ENTER DISPLACEMENT FOR SEGMENT NO. 3 (mm) : *15.*

(6) The lift and rotation angle of cam for each motion segment, computed by the system are as follows.

MOTION CURVE	ROTATION ANGLE (degree)	DISPLACEMENT (mm)
CM_1	88.11954	20.00
SH_2	51.88041	15.00
SH_4	51.88041	15.00
CM_5	88.11954	20.00

(7) The system displays displacement, velocity and acceleration diagrams. The user can choose any one of the following options.

DISPLAY DISPLACEMENT CURVE	–	1
DISPLAY VELOCITY CURVE	–	2
DISPLAY ACCELERATION CURVE	–	3
QUIT	–	Q

ENTER THE OPTION : *Q*

(8) The user, at this stage is allowed to change the motion specifications.

DO YOU WANT TO CHANGE THE SPECIFICATIONS (Y/N) ? : *N*

If the user enters Y, steps 1 to 10 are repeated.

(9) Enter the data about follower type.

ENTER BASE CIRCLE RADIUS OF CAM (mm) : *50.*

ENTER FOLLOWER ROLLER RADIUS (mm) : *10.*

CUTTER RADIUS (mm) : *15.*

(10) The system displays cam profile and cutter path for visual inspection. Fig5.10and Fig.5.11show cam profile, cutter path and cutter in incremental positions.

(11)

MAX. PRESSURE ANGLE (RISE) : 19.76^0 AT CAM ANGLE : 74.00^0

MAX. PRESSURE ANGLE (FALL) : 19.76^0 AT CAM ANGLE : 186.18^0

(12) The list of material combinations for cam and the follower [Table 3.2] will appear on the screen. The user has to enter the code for combination.

ENTER CODE FOR METAL COMBINATION : *E*

ENTER TOTAL LOAD ON FOLLOWER (N) : *2900*

ENTER FOLLOWER ROLLER WIDTH (mm) : *15.*

MAX. CONTACT STRESS : 734.222 MPa AT CAM ANGLE : 89.83^0

(13) Optimum cam size for precribed pressure angle.

ENTER PERMISSIBLE MAX. PRESSURE ANGLE FOR RISE : 30^0

ENTER PERMISSIBLE MAX. PRESSURE ANGLE FOR FALL : 30^0

MIN. BASE CIRCLE RADIUS OF CAM TO AVOID FOLLOWER JAMMING (mm) : 22.66

(14) The user, at this stage is allowed to change the cam-follower parameters.

DO YOU WANT TO CHANGE THE CAM PARAMETERS (Y/N)? : *Y*

(15)

ENTER BASE CIRCLE RADIUS (mm) : *23.*

ENTER FOLLOWER ROLLER RADIUS (mm) : *10.*

CUTTER RADIUS (mm) : *15.*

MAX. PRESSURE ANGLE (RISE) : 29.34^0 AT CAM ANGLE : 68.00^0

MAX. PRESSURE ANGLE (FALL) : 29.34^0 AT CAM ANGLE : 192.17^0

CONTACT STRESS EXCEEDS PERMISSIBLE VALUE

MAX. CONTACT STRESS : 795.287 MPa AT CAM ANGLE : 260.00^0

Contact stress is exceeding max. permissible value.

DO YOU WANT TO CHANGE THE CAM PARAMETERS (Y/N)? : *Y*

(16)

ENTER BASE CIRCLE RADIUS (mm) : *30.*

ENTER FOLLOWER ROLLER RADIUS (mm) : *10.*

CUTTER RADIUS (mm) : *15.*

MAX. PRESSURE ANGLE (RISE) : 26.08^0 AT CAM ANGLE : 70.00^0

MAX. PRESSURE ANGLE (FALL) : 26.08^0 AT CAM ANGLE : 190.19^0

MAX. CONTACT STRESS : 770.136 MPa AT CAM ANGLE : 174.00^0

DO YOU WANT TO CHANGE THE CAM PARAMETERS (Y/N)? : *N*

(17)

DO YOU WANT NC CODE FOR THE CAM (Y/N)? : *Y*

If the user enters Y, the system generates NC code. The NC code of a cam for this example is given in the appendix.

(18)

DO YOU WANT TO DESIGN ANOTHER CAM (Y/N)? : *Y*

If the user enters Y, steps 1 to 18 are repeated.

Final Output

RADIALLY TRANSLATING ROLLER FOLLOWER

MOTION CURVE	ROTATION ANGLE (degree)	DISPLACEMENT (mm)
CM_1	88.11954	20.00
SH_2	51.88041	15.00
SH_4	51.88041	15.00
CM_5	88.11954	20.00

BASE CIRCLE RADIUS OF CAM (mm) : *30.*

RADIUS OF FOLLOWER ROLLER (mm) : *10.*

CUTTER RADIUS (mm) : *15.*

CAM MATERIAL : MOLYBDENUM CAST IRON - HOT QUENCHED

FOLLOWER MATERIAL : HARDENED AND GROUND STEEL

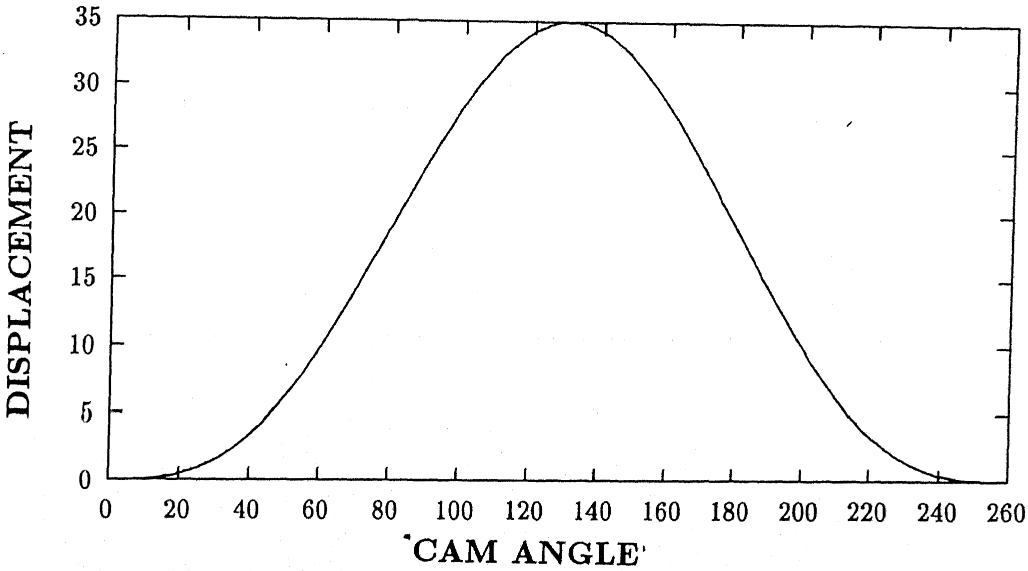


Figure 5.2: Displacement curve of example - 1

FOLLOWER WIDTH (mm) : *15.*

TOTAL LOAD ON FOLLOWER (N) : *2900*

MAX. PRESSURE ANGLE (RISE) : 26.08^0 **AT CAM ANGLE :** 70.00^0

MAX. PRESSURE ANGLE (FALL) : 26.08^0 **AT CAM ANGLE :** 190.19^0

MAX. CONTACT STRESS : 734.222 MPa AT CAM ANGLE : 89.83^0

The displacement, velocity and accleration diagrams are shown in Fig 5.2, . Fig 5.3 and Fig 5.4 respectively.

NC code for this example is given in the appendix.

5.1.2 Example - 2

A cam is to be operated with translating flat faced follower. The follower moves outwards 30 mm with 3-4-5 polynomial motion, while cam rotates through 100^0. The return of the follower takes place by cycloidal motion for 90^0 of cam rotation

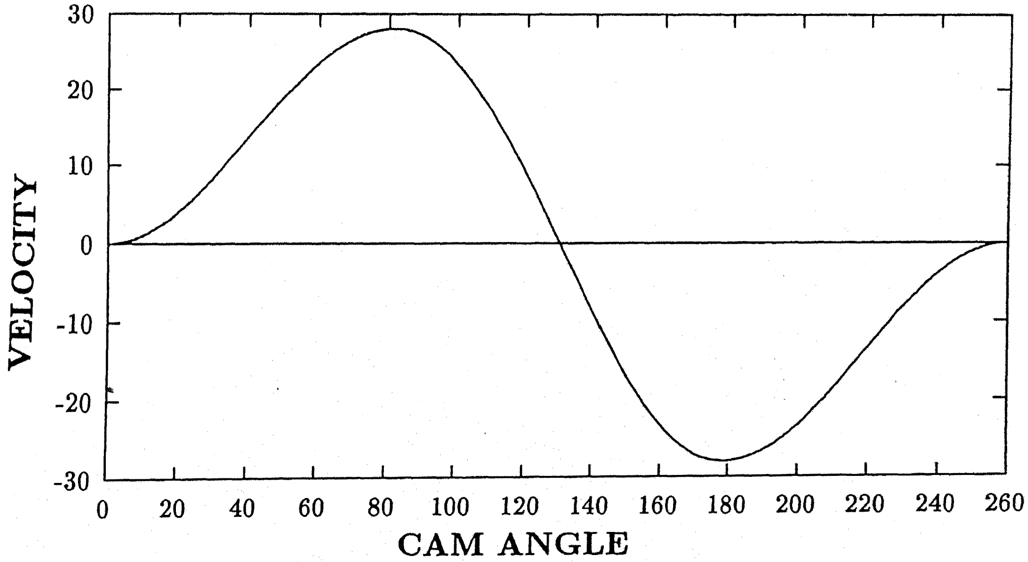


Figure 5.3: Velocity curve of example - 1

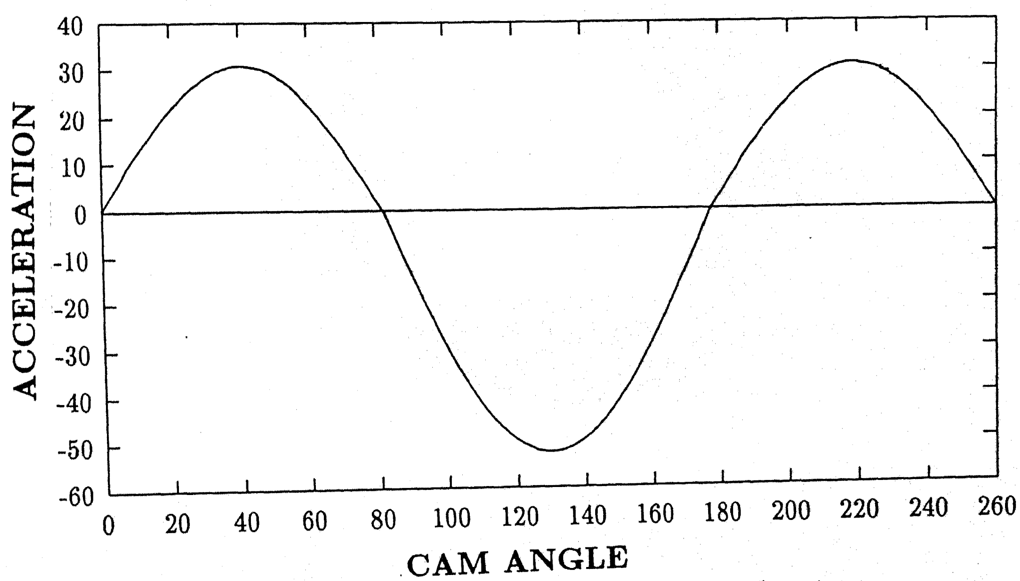


Figure 5.4: Acceleration curve of example - 1

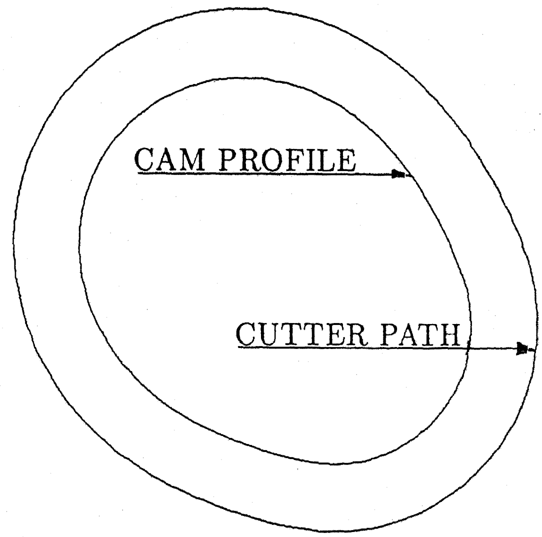


Figure 5.5: Cam profile and Cutter path for example - 1

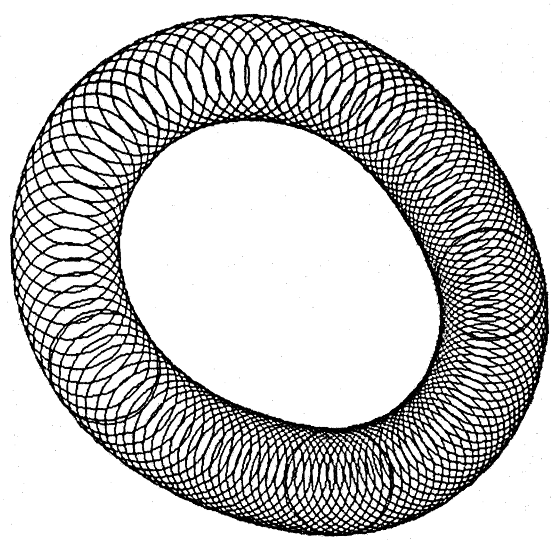


Figure 5.6: Incremental positions of Cutter for example - 1

angle. The follower remains steady between rise and return for 90^0 of cam angle. The cam and the follower both are made of steel. Let the base circle of cam be 60 mm and total load on the follower 3000 N. The width of the follower is 15 mm and radius of the cutter to be used to manufacture a cam, is 15 mm.

Trial Run of Example - 2

(1)

TYPES OF CAM FOLLOWER CONFIGURATIONS		CODES
FLAT-FACED TRANSLATING FOLLOWER	–	1
RADIALLY TRANSLATING ROLLER FOLLOWER	–	2
OFFSET TRANSLATING ROLLER FOLLOWER	–	3
SWINGING ROLLER FOLLOWER	–	4
CENTRIC SWINGING FLAT-FACED FOLLOWER	–	5
ECCENTRIC SWINGING FLAT-FACED FOLLOWER	–	6

ENTER THE FOLLOWER TYPE : *1*

(2)

ENTER TOTAL FOLLOWER DISPLACEMENT (mm) : *30.*

ENTER TOTAL CAM ROTATION ANGLE (degree) : *280.*

ENTER NO. OF CURVE SEGMENTS : *3*

(3)

DO YOU WANT HELP ON CURVE SELECTION (Y/N) ? : *Y*

If user enters yes, Table 2.2 and Table 4.1 will appear on the screen. From these tables user will get help in selecting the motion curves.

(4)

ENTER THE MOTION CURVE FOR SEGMENT NO. 1 : *5P_3*

ENTER THE MOTION CURVE FOR SEGMENT NO. 2 : *Dwel*

ENTER THE MOTION CURVE FOR SEGMENT NO. 3 : *CM_6*

(5)

ENTER CAM ROTATION ANGLE FOR RISE (degree) : *100*

ENTER CAM ROTATION ANGLE FOR FALL (degree) : *90*

(6)

MOTION CURVE	ROTATION ANGLE (degree)	DISPLACEMENT (mm)
5P_3	100.00	30.00
Dwel	90.00	00.00
CM_6	90.00	30.00

(7)

DISPLAY DISPLACEMENT CURVE	–	1
DISPLAY VELOCITY CURVE	–	2
DISPLAY ACCELERATION CURVE	–	3
QUIT	–	Q

ENTER THE OPTION : *Q*

(8)

DO YOU WANT TO CHANGE THE SPECIFICATIONS (Y/N) ? : *N*

(9)

ENTER BASE CIRCLE RADIUS OF CAM (mm) : *60.*

CUTTER RADIUS (mm) : *15.*

(10) The system displays cam profile and cutter path for visual inspection. Fig 5.5 and Fig. 5.6 show cam profile, cutter path and cutter in incremental positions.

(11) The list of material combinations for cam and the follower [Table 3.2] will appear on the screen. The user has to enter the code for combination.

ENTER CODE FOR METAL COMBINATION : *F*

ENTER TOTAL LOAD ON FOLLOWER (N) : *3000*

ENTER FOLLOWER ROLLER WIDTH (mm) : *15.*

MAX. CONTACT STRESS : 819.564026 MPa AT CAM ANGLE : 214^0

(12)

MIN. BASE CIRCLE RADIUS OF CAM TO AVOID UNDRE-
CUTTING (mm) : 50.

OPTIMUM LENGTH OF FLAT FACE (mm) : 64.45

(13)

DO YOU WANT TO CHANGE THE CAM PARAMETERS (Y/N)? : *Y*

(14)

ENTER BASE CIRCLE RADIUS (mm) : *50.*

CUTTER RADIUS (mm) : *15.*

CONTACT STRESS EXCEEDS PERMISSIBLE VALUE

MAX. CONTACT STRESS : 3060.27 MPa AT CAM ANGLE : 214.00^0

DO YOU WANT TO CHANGE THE CAM PARAMETERS (Y/N)? : *Y*

(15)

ENTER BASE CIRCLE RADIUS (mm) : *53.*

CUTTER RADIUS (mm) : *15.*

MAX. CONTACT STRESS : 1384.91 MPa AT CAM ANGLE : 214.00^0

DO YOU WANT TO CHANGE THE CAM PARAMETERS (Y/N)? : *N*

(16)

DO YOU WANT NC CODE FOR THE CAM (Y/N)? : *Y*

If the user enters Y, the system generates NC code. The NC code of a cam for this example is given in the appendix.

(17)

DO YOU WANT TO DESIGN ANOTHER CAM (Y/N)? : *Y*

If the user enters Y, steps 1 to 17 are repeated.

Final Output

TRANSLATING FLAT-FACED FOLLOWER

MOTION CURVE	ROTATION ANGLE (degree)	DISPLACEMENT (mm)
5P_3	100.00	30.00
Dwel	90.00	00.00
CM_6	90.00	30.00

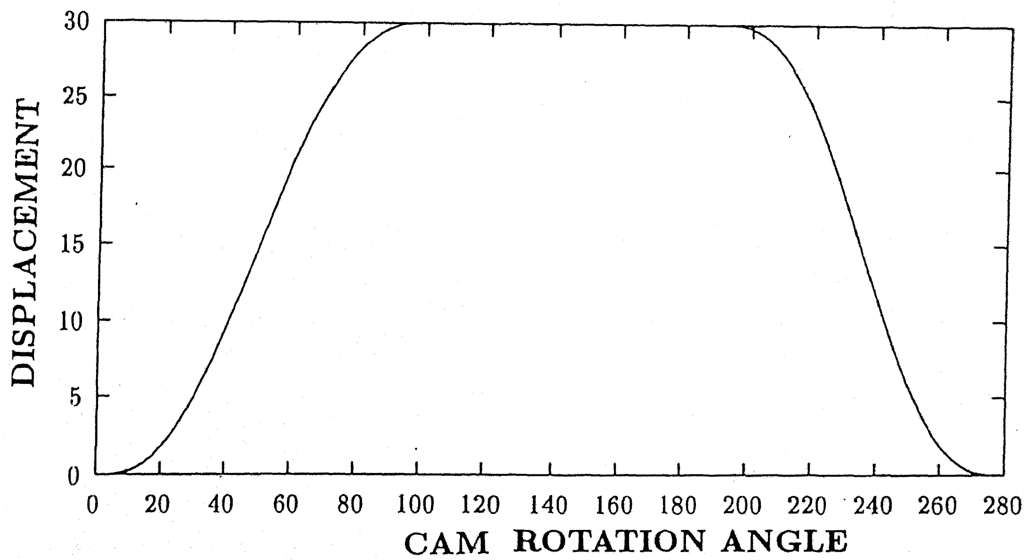


Figure 5.7: Displacement curve of example - 2

BASE CIRCLE RADIUS OF CAM (mm) : *53.*

OPTIMUM LENGTH OF FLAT FACE (mm) : 64.45

CUTTER RADIUS (mm) : *15.*

FOLLOWER MATERIAL : 1.05% C, TOOL STEEL, RC 60 - 63

CAM MATERIAL : LOW CARBON STEEL, RC 56 - 61

FOLLOWER WIDTH (mm) : *15.*

TOTAL LOAD ON FOLLOWER (N) : *3000*

MAX. CONTACT STRESS : 1384.911 MPa AT CAM ANGLE : 214^0

The displacement, velocity and accleration diagrams are shown in Fig 5.7, . Fig 5.8 and Fig 5.9 respectively.

NC code for this example is given in the appendix.

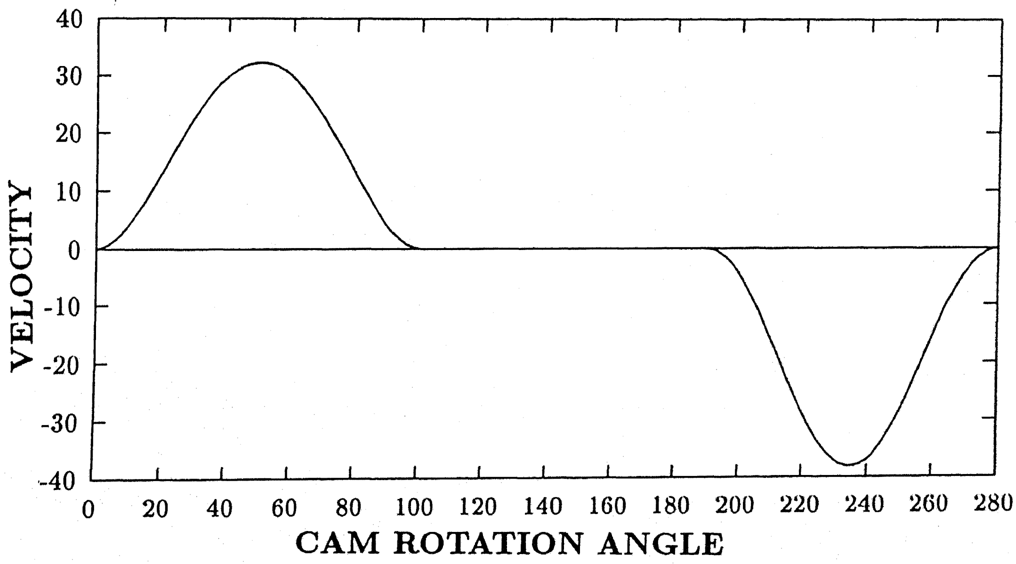


Figure 5.8: Velocity curve of example - 2

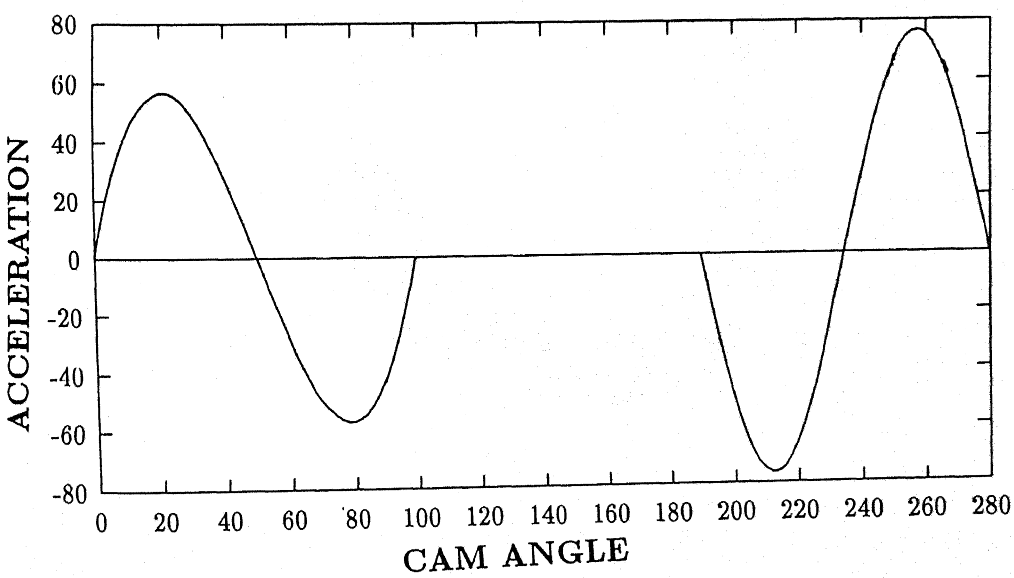


Figure 5.9: Acceleration curve of example - 2

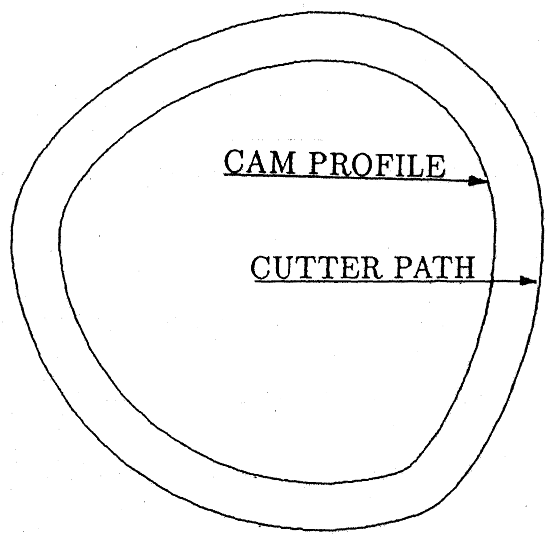


Figure 5.10: Cam profile and Cutter path for example - 2

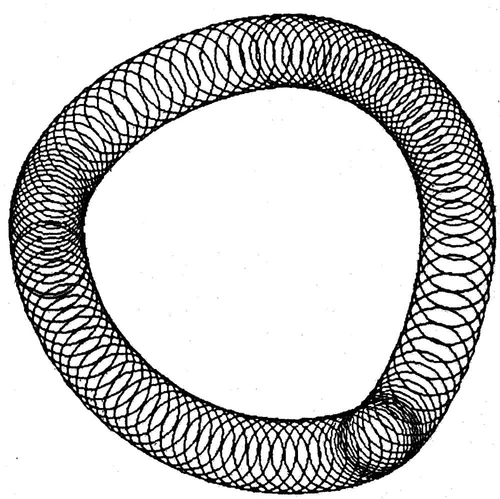


Figure 5.11: Incremental positions of Cutter for example - 2

5.2 Discussion

The developed package acts as an excellent tool in designing the plate cams to the specified limitations of pressure angle, radius of curvature and contact stress between cam and follower interactively. The system displays the displacement, velocity and acceleration diagrams, as well as cam profile and cutter path for visual inspection. User can take decisions at critical stages of design. Values of parameters such as base circle radius, follower radius, rotation angle can be changed and their effects seen immediatly on design.

The graphical display permits more design iterations per unit time. The integration of design and manufacturing results in reduced product developement time because of additional design and analysis cycle.

Chapter 6

Conclusions

6.1 Technical Summary

A generalised procedure for the analysis and design of plate cams has been developed in the present thesis work. The scope of this thesis work is limited to cam and follower confugrations including radially translating roller follower, offset translating roller follower, swinging roller follower, centric oscillating flat-faced follower, eccentric oscillating flat-faced follower. A wide selection of motion curves permits the designer to specify any described motion of operating element by synthesizing several curves into a displacement diagram. The software package developed acts as an excellent tool in designing the optimum plate cams to the specified limitations of pressure angle, radius of curvature and contact stress between cam and follower.

The NC code generated is a simple one, consisting of linear interpolation, rapid feed and coordinate setting commands. It does not include canned cycles. G and M codes depends upon the type of controller used within the machine. In

the software program developed, these codes are designed for Denford Milling machine with Fanuc controller.

6.2 Scope for Future Work

The present work focuses on the design and manufacturing of plate cams for six types of cam follower configurations. Optimization of cams with offset translating roller, swinging roller follower and oscillating flat-faced follower is not considered in the present work. It requires solution of complicated equations and cummrcial packages to solve these equations. The theory of envelops can be extended to the design of cams with irregular followers, however it requires detailed investigation of follower geometry. Selection of roller followers which are generally ball bearings or needle bearings can be incorporated in the program. A table of load capacities and geometries for standard for standard roller followers, extracted from the manufacturer's catalog can be incorporated in the program.

The analysis of frictional forces is not considered since the frictional coefficients given in the handbooks are not useful for a particular application. The conditions under which values are obtained are usually not given. Spring which keeps follower in contact with the cam can be designed if large data about spring material and stress factors is available. Dynamic analysis of cam follower mechanism is not considered in the present work. Dynamic characteristics such as frequency responce of the system, comes under the field of vibrations, and is beyond scope of this work.

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Appendix

Partial listing of NC code generated by the system for example - 1 is giev below.

```
O2223
G28 X0 Y0 Z0;
G90 G92 X0 Y0 Z0;
S600 M03;
G00 X0 Y0 Z0;
G01 X0 Y40.00 F100;
G01 Z-2;
M98 P1400;
G01 Z-4;
M98 P1400;
G01 Z-6;
M98 P1400;
G01 Z-8;
M98 P1400;
G01 Z-10;
```

```
M98 P1400;
G01 Z-12;
M98 P1400;
G01 Z-14;
M98 P1400;
M30;

.

O1400;
G01 X1.396000 Y39.976109;
G01 X2.790530 Y39.906391;
G01 X4.182490 Y39.793739;
G01 X5.571180 Y39.641029;
G01 X6.956280 Y39.451012;
G01 X8.337810 Y39.226330;
G01 X9.716170 Y38.969421;
G01 X11.092020 Y38.682480;
G01 X12.466350 Y38.367470;
G01 X13.840330 Y38.026009;
G01 X15.215390 Y37.659420;
G01 X16.593069 Y37.268631;
G01 X17.975010 Y36.854229;
G01 X19.362940 Y36.416389;
```

```
G01 X20.758570 Y35.954899;

G01 X22.163580 Y35.469139;

G01 X23.579540 Y34.958099;

G01 X25.007879 Y34.420391;

G01 X26.449829 Y33.854240;

G01 X27.906389 Y33.257542;

G01 X29.378260 Y32.627861;

G01 X30.865820 Y31.962490;

G01 X32.369080 Y31.258459;

G01 X33.887661 Y30.512590;

G01 X35.420761 Y29.721550;

G01 X36.967121 Y28.881880;

.

.

.

G01 X0.000010 Y40.000000;

M99;
```

Partial listing of NC code generated by the system for example - 2 is giev below.

```
O2222
```

```
G28 X0 Y0 Z0;
G90 G92 X0 Y0 Z0;
S600 M03;
G00 X0 Y0 Z0;
G01 X68.00 Y0 F100;
G01 Z-2;
M98 P1400;
G01 Z-4;
M98 P1400;
G01 Z-6;
M98 P1400;
G01 Z-8;
M98 P1400;
G01 Z-10;
M98 P1400;
G01 Z-12;
M98 P1400;
G01 Z-14;
M98 P1400;
M30;
O1500;
G01 X67.953987 Y2.571220;
G01 X67.799339 Y5.503220;
```

G01 X67.514816 Y8.745430;

G01 X67.083908 Y12.248800;

G01 X66.494522 Y15.966070;

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G01 X63.714199 Y27.959709;

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G01 X61.016918 Y36.256500;

G01 X59.430710 Y40.388550;

G01 X57.698811 Y44.468700;

G01 X55.833191 Y48.469730;

G01 X53.847408 Y52.367378;

G01 X51.756321 Y56.140221;

G01 X49.575779 Y59.769760;

G01 X47.322300 Y63.240360;

G01 X45.012829 Y66.539223;

G01 X42.664440 Y69.656273;

G01 X40.294060 Y72.584106;

G01 X37.918190 Y75.317886;

G01 X35.552711 Y77.855217;

G01 X33.212601 Y80.195992;

G01 X30.911730 Y82.342247;

G01 X28.662621 Y84.298012;

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G01 X20.377380 Y90.355042;
G01 X18.515600 Y91.474220;
G01 X16.742849 Y92.457336;
G01 X15.057660 Y93.316406;
G01 X13.456110 Y94.063606;
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